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BACKGAMMON
How Much Should You Bet ?

BY: MICHELIN CHABOT

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ACKNOWLEDGMENTS

I give particular thanks to the publisher "Academic Press Inc." for having given permission to use certain extracts and formulas originally found in the book:

"The Theory of Gambling and Statistical Logic", (1977)

by Richard A. Epstein, without which this publication would not have been possible. These formulas and extracts are well identified in this publication.

I would like to give special thanks to all my friends who read the manuscript, especially to Paul Stebbing, Narvey Goldman, Réginald Proulx and "A. Oppenheim", for their assistance; and to my wife, Lise, for her constant encouragement.

Michelin Chabot

FOREWORD

A few years ago, after having read all the books (available on the market) on backgammon, I realized that, practically speaking, none of the books could help me to answer the following question:

- How much should I bet?

This question might also be formulated in one of the following ways:

- What is my appropriate bet?
- What variables must be considered to fix my bet?

After months of research reading several books on money management, I came to the conclusion that there are two main types of books on the subject:

1) Books which give a "miracle approach" on how to win. These are not difficult to understand, but some of the advice given is of doubtful character.

2) Books filled with so many advanced mathematical formulas that only a mathematician can decipher them.

Because the latter group seemed to offer the correct approach, I decided, for personal reasons (and not for purposes of publication) to summarize the most important formulas and concepts. The typed summary finally came to a total of 30 pages. Some of my friends were interested in reading this summary, but didn't understand it very well, even though it seemed very clear to me. Therefore, I gave them explanations of certain points, and discovered that they were very interested in the subject. Their interest motivated me to transform the 30 page draft into a text for publication. As a result, I added the explanations given to friends along with more practical examples on how to apply the formulas. A short while later, the text expanded to 70 pages. The title which then came to mind was: "Backgammon: Principles of Money Management".

I decided that, before publication, it would be advisable to seek out the comments of Narvey Goldman, whom I consider to be both an expert on backgammon and a friend. He made the following main criticisms:

1) The text is too impersonal.

2) The text contains no backgammon diagrams (positions).

3) The text is too mathematical in character, even if the numerous examples enable a sufficient understanding of the concepts to be grasped.

In regard to the first criticism, I brought to his attention that, being an engineer, it was normal for me to treat such subjects impersonally. To render the text more personal and lively, I added fictitious characters to the text. John is the student who wants to know everything about the principles of money management, and especially, about how much he should bet while playing backgammon. Peter is the mathematician who explains with a practical approach, step by step, everything John wants to know.

To remedy the lack of backgammon diagrams, I included a chapter entitled "Money Management Versus The Doubling Cube Theory".

To offset the highly mathematical character of the text, I removed elements which might be considered non-essential and added a new chapter entitled "Principles of Money Management in Practice". This chapter, which contains a minimum of mathematical content, was the easiest to write and, paradoxically, will undoubtedly be the most enjoyable and profitable chapter for the reader.

When reading the manuscript, some of my friends suggested that I begin the book with what is presently Chapter six. I felt, however, that in order to provide the basic principles, it was necessary to keep the original order, even though the opening chapters are mathematical in character. One of my friends thought it would be better to eliminate the opening chapters entirely, but I pointed out to him that if these chapters were missing, the criticism might be made that the later chapters were without a solid foundation.

Although the title selected is "Backgammon - How much should You bet?", and the word "you" is never used in the text, I sincerely believe that the title is not misleading, because, once having read the book, the reader should be able to establish for himself how much <u>he</u> should bet.

If, after reading the publication, the reader has any comments to make, questions to ask, or points to elaborate on, I invite him to write me at the address given below, and I would be more than pleased to answer him.

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INTRODUCTION

The goal of this publication, entitled "Backgammon - How much should you bet?" is to explain the principles of money management and the variables that a backgammon player must consider in determining how much he should bet. This will be done by studying two fictitious players, John Stone and Peter Brown.

John has been playing backgammon for five years. He has read the better known books on the subject and believes that he plays the game well. His current betting system involves dividing the amount of money in his pocket by 50 to arrive at the amount per point that he is willing to risk. John has learned, however, that some supposedly strong backgammon players have lost a lot of money betting in this fashion or by some other system. The reason for their losses, he also learned, is that they had not bet in accordance with the basic principles of money management. Hoping to avoid their mistakes, John decided to learn more about these important rules. Not being a mathematician himself, he was somewhat intimidated by all the formulas he would have to study. Still, he's confident that, with the right person to patiently explain it all using a set of examples, he could grasp the principles of money management.

His research brought him into touch with Peter Brown who agreed to help John learn how to determine how much to bet at backgammon. Peter is every bit as strong a backgammon player as John, but with the advantage of a solid mathematical background. Peter has already read and understood the principles of money management for games in general.

Backgammon is one particular game, and Peter first explains that the principles that apply to any specific game should follow the same basic principles that apply for games in general. **The first step, then, must be to master money management for games in general and, the second step, to make some adjustments for the game of backgammon.**

Peter begins by outlining the following concepts:

- the concept of expectation (Chapter 1);
- the concept of hourly expectation (Chapter 1);
- the concept of single-trial probability of success (Chapter 1);
- the concept of probability of success (Chapter 2);
- the concept of probable number of games to play (Chapter 3);

Once John has understood these concepts, Peter explains the possible criteria to be used when establishing a bet (Chapter 4) and follows by assessing strategy for games in general (Chapter 5).

Once he has assimilated these principles that apply to all games, Peter reviews the adjustments to be made for backgammon (Chapter 6) and explains the suggested approach for a backgammon player who hopes to make the appropriate bet (Chapter 7). Peter and John study the relationship between the principles of money management and the Doubling Cube Theory (Chapter 8). Finally, Peter explains how John should proceed in practical terms to assure himself that he is correctly adhering to the principles of money management (Chapter 9).

Encouraged and excited by their plans, John tells Peter, "Let's get going ! "

CHAPTER 1 - CALCULATION OF THE EXPECTATION, HOURLY EXPECTATION AND SINGLE-TRIAL PROBABILITY OF SUCCESS

Peter explains that, before being able to elaborate the principles of money management, it is necessary to determine if a game is favorable or not. That means calculating the expectation. The expectation is, in fact, the theoretical gain average or the theoretical loss average that a player should obtain only after having played the game a large number of times. If the expectation is positive, then the game is favorable; if the expectation is nil, then the game is fair; and if the expectation is negative, then the game is unfavorable. In a general manner, the expectation is calculated as follows:

$$E = \sum_{i=1}^{N} P_i R_i = P_1 R_1 + P_2 R_2 + P_3 R_3 + \dots + P_n R_n \qquad \text{Formula 1}$$

E = Expectation

N = Number of possibilities

P_i = probability of occurrence of the possibility i

R_i = Result associated with the possibility i

The symbol Σ means the sum, the symbol $\sum_{i=1}^{N}$ means the sum of the elements from i=1 to i=N and the expression $\sum_{i=1}^{N} P_i R_i$ means the sum of all the products $P_i R_i$, i varies from 1 and goes until N. Peter points out that the sum of the probability of occurrence of each possibility should always be equal to 1.0. John has already found this first formula to be too complex for him, but Peter assures him that, if he can understand the following examples, he can also grasp the meaning of the formula above.

Example 1:

Dick Butters plays American roulette. There are 38 possibilities, 18 red, 18 black, 0 and 00. Dick bets one unit on one color. What is his expectation?

For this game, there are 2 possibilities, namely:

1) winning 1 unit with a probability of 18/38, (for this first possibility, $P_1 = 18/38$, $R_1 = +1$); or

2) losing 1 unit with a probability of 20/38, (for this second possibility, $P_2 = 20/38$, $R_2 = -1$).

The expectation is calculated as follows:

$$E = \sum_{i=1}^{2} P_i R_i = P_1 R_1 + P_2 R_2$$

$$E = (18/38) \times 1 + (20/38) \times (-1) = -2/38 = -5.26\% \text{ of the unit risked}$$

For this game then, the player's expectation is -5.26%. John still does not understand perfectly. Peter explains to him that, if this player gambles $1 for 1,000,000 games, theoretically he should win 18/38 x 1,000,000 games = 473,684 games or $473,684 and lose 20/38 = 526,316 games or $526,316; for a loss of ($526,316 - $473,684 =) $52,632. If the player should lose $52,632 on 1,000,000 games at $1 a game, this represents 5.26% of the total amount bet. If the player's expectation is -5.26%, then the casino's advantage is 5.26%. If the unit bet is $10, it follows that the gambler is going to lose (on average) 5.26% x $10 = $0.53 per game. Because John is a backgammon player, Peter decides to offer examples involving dice.

Example 2:

Dick now plays the following game: one die is thrown and, if the result is 6, he wins $5. On the other hand, if the result is 1, 2, 3, 4 or 5, he loses $1. What is the expectation?

For this game, there are two possibilities, namely:

1) winning 5 units with a probability of 1/6, ($P_1 = 1/6$, $R_1 = +5$); or

2) losing 1 unit with a probability of 5/6, ($P_2 = 5/6$, $R_2 = -1$).

The expectation is calculated as follows:

$$E = \sum_{i=1}^{2} P_i R_i = P_1 R_1 + P_2 R_2$$

$$E = 1/6 \times 5 + 5/6 \times (-1) = 0\% \text{ of the unit risked}$$

Peter explains that since E = 0%, this game is the same as playing heads or tails and, consequently, it is a fair game.

Example 3:

Dick plays yet another game: one die is thrown; if the result is 1, 2, 3 or 4 he loses 1 unit; if the result is 5, he wins 2 units and, if the result is 6, then he wins 3 units. What is the expectation?

For this game, there are three possibilities, namely:

1) lose 1 unit with a probability of 4/6 ($P_1 = 4/6$, $R_1 = -1$);
2) win 2 units with a probability of 1/6 ($P_2 = 1/6$, $R_2 = 2$); or
3) win 3 units with a probability of 1/6 ($P_3 = 1/6$, $R_3 = 3$).

The expectation is calculated as follows:

$$E = \sum_{i=1}^{3} P_i R_i = P_1R_1 + P_2R_2 + P_3R_3$$

$$E = 4/6 \times (-1) + 1/6 \times 2 + 1/6 \times 3 = 1/6 = 16.67\% \text{ of the unit risked}$$

This is a favorable game as the player will win an average of $0.17 for each dollar bet.

* * * * *

Now that John knows how to calculate the expectation, Peter gives him the formula for calculating the hourly expectation.

Hourly expectation = Average Bet x Expectation x Games per Hour Formula 2

Peter notes that, if a player wishes to increase his hourly expectation, he must increase one or several of the variables in the formula given above. Peter also points out that, in order to maximize the expectation, a player should try to maximize the number of hours to be played. In other words, when the game is favorable (i.e. when the expectation is positive), the longer the playing time, the greater the expectation will be.

Example 4:

Dick plays a game in which his average bet is \$10. His expectation is 1% (or 0.01) of the unit gambled on each game. Dick plays on average of 50 games an hour. What is his hourly expectation?

Using Formula 2, we get the following:

Hourly expectation = Average bet x Expectation x Games per Hour

Hourly expectation = \$10 x 1% x 50 = \$5 /hour

His hourly expectation should thus be \$5 per hour. If Dick increases one or several variables (average bet, or expectation or games per hour), his hourly expectation will also rise. If Dick increases the number of hours to be played, he will by the same token, increase his expectation.

* * * * *

Now John is able to calculate the expectation and the hourly expectation for games that have a strictly mathematical character. Peter explains that, when the expectation of a game is known, it is possible to evaluate the single-trial probability of success defined as "the probability of occurrence renormalized with respect to an even payoff" (The Theory of Gambling and Statistical Logic, page 113). This concept will prove very useful in understanding the significance of the formulas to be outlined in the chapters to come. Knowing "E", the expectation of a game, it is possible to calculate "P" the single-trial probability of success, in the following manner:

P = .5 + E/2 Formula 3

P = single-trial probability of success

E = Expectation

Peter points out that if a game is unfavorable, then the expectation is negative and the single-trial probability of success is inferior to 50%; if a game is fair, then the expectation is nil and the single-trial probability of success is equal to 50%; and if a game is favorable, the expectation is positive and the single-trial probability of success is superior to 50%. Peter gives the following examples to illustrate.

Example 5:

The expectations obtained for examples 1, 2 and 3 are respectively -5.26%, 0% and 16.67% of the units risked. What is the single-trial probability of success for each of these examples?

In example 1, E = -5.26% of the units risked and by using Formula 3, the single-trial probability of success is:

$P = .5 + E/2 = .5 + (-5.26\%)/2 = 47.37\%$; not a favorable game.

In example 2, the expectation is nil and, by using Formula 3, the single-trial probability of success is 50%; this is a fair game.

In example 3, the expectation is 16.67% of a unit risked and the single-trial probability of success is calculated as follows:

$P = .5 + E/2 = .5 + (16.67\%)/2 = 58.33\%$; this game is favorable.

* * * * *

Example 6:

Peter shows John the following extract from the book "The Theory of Gambling and Statistical Logic", pages 150 and 151 concerning the game of Chuck-A-Luck:

> "Three dice are agitated inside a double-ended, rotatable cage with an hourglass cross section (the "Bird Cage"). A player may wager upon any of the outcomes 1 through 6. If one (and only one) die exhibits that outcome (p = 75/216), the player wins at even odds; if two dice exhibit that outcome (p = 15/216), the payoff is 2 to 1; if all three dice show the player's choice (p = 1/216), the payoff is 3 to 1; otherwise (that is, if the specified outcome appears on none of the three dice) the player loses. Elementary calculations indicate a probability of success of 0.461."

Peter asks John if he could prove that this percentage of 46.1% is exact? What is the hourly expectation for a player who plays with a bet of $5 with ten games per minute?

John first makes a list of the 4 possibilities:

1) win 1 unit with a probability of 75/216, (P_1 = 75/216, R_1 = +1);

2) win 2 units with a probability of 15/216, (P_2 = 15/216, R_2 = +2);

3) win 3 units with a probability of 1/216, (P_3 = 1/216, R_3 = +3); or

4) lose 1 unit with a probability of 125/216, (P_4 = 125/216, R_4 = -1).

He then calculates the expectation using Formula 1:

$$E = \sum_{i=1}^{4} P_i R_i = P_1R_1 + P_2R_2 + P_3R_3 + P_4R_4$$

$$E = (75/216) \times 1 + (15/216) \times 2 + (1/216) \times 3 + (125/216) \times (-1)$$

$$E = -17/216 = -7.87\% \text{ of the unit risked}$$

John calculates the hourly expectation by using Formula 2.

Hourly expectation = Average bet x Expectation x Games per hour

Hourly expectation = \$5 x (-7.87%) x 600 games per hour = - \$236.10

Knowing "E", John then calculates "P" using Formula 3:

$$P = .5 + E/2 = .5 + (-7.87\%)/2 = 0.46065$$

Therefore, John concludes that the single-trial probability of success is, in effect, 46.1%, and that the hourly expectation (with a bet of \$5 and with ten games per minute) is - \$236.10/hour.

* * * * *

John now knows that, for games of a strictly mathematical character, the calculation of expectation, the hourly expectation and the calculation of the single-trial probability of success can be established using a mathematical approach. However, Peter points out that for certain games of skill and chance, such as blackjack and backgammon, it is necessary to use methods of a statistical nature. If a player plays a game in which the probability of winning is 50%, (heads or tails, for example), then the results obtained from a number of attempts can be evaluated statistically. Table 1 verifies this with the following results.

Table 1

An example of the results obtained with the help of statistics for a game where P = 50%

Number of attempts	95% of the time, the proportion obtained will be between... and...	95% of the time the margin of error will be inferior to ...
10	19% and 81%	31%
30	32% and 68%	18%
100	40% and 60%	10%
200	43% and 57%	7%
500	46% and 54%	4%
1 000	47% and 53%	3%

From the figures in table 1 Peter presents Diagram 1.

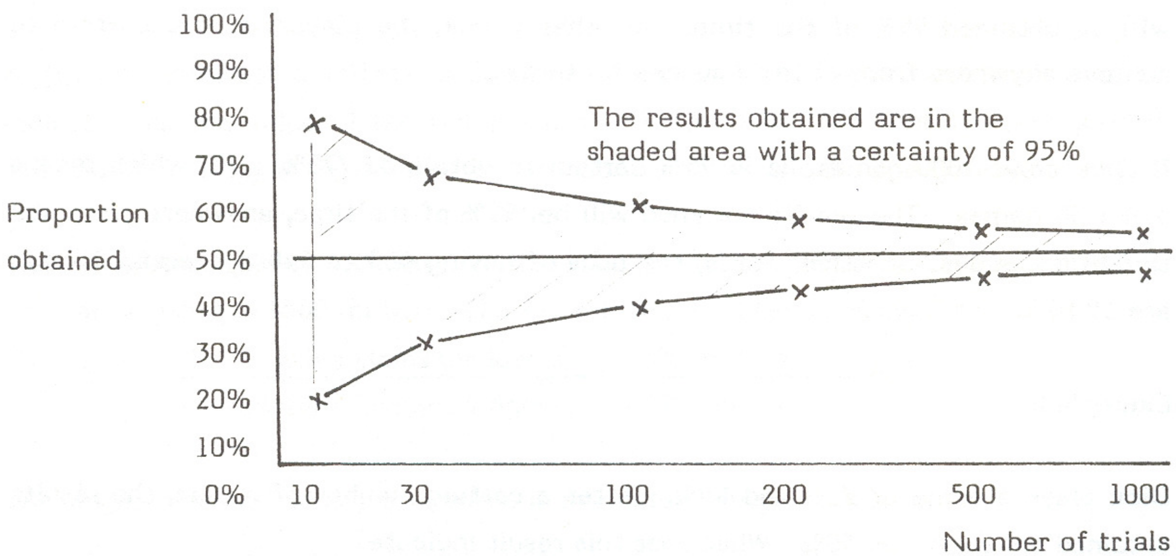


Diagram 1: Illustration of statistical results for a game where P = 50%

Peter explains that Diagram 1 is a practical illustration of the law of large numbers which can be expressed as follows: "The probability of moving towards a true theoretical proportion increases with the number of attempts". This law makes it possible to evaluate the accuracy of a result in relation to the number of attempts (trials). In other words, if a proportion is unknown, and a gambler plays a large number of trials, it follows that the precision of the proportion obtained increases with the number of attempts. The larger the number of trials, the more precise the result will be. John admits that it seems logical. To be sure that John understands the law of large numbers (also called the law of averages), Peter gives him the following examples.

Example 7:

Dick plays the following game: he throws four dice and tries to obtain a 6 (the probability of obtaining a desired number by shaking four dice is $1 - (5/6)^4 = 51.775\%$). What will the probable number of successful throws be if he plays 100 or 1000 games?

If Dick plays 100 games, the theoretical proportion, with a margin of error of 10%, will be obtained 95% of the time. In other words, the player is 95% certain to achieve anywhere from 42 to 62 successful throws.

If Dick plays 1000 games, he is 95% certain to obtain 51.775% $\pm$ 3% which means 518 $\pm$ 30 games. The results obtained will be, 95% of the time, anywhere from 488 to 548 successes. In other words, the odds of having 488 to 548 successful throws are 19 to 1.

Example 8:

Dick plays a game of skill and luck. After a certain number of games, the results obtained are even, i.e. 50%. What does this result indicate?

If 10 games have been played, the margin of error is less than 31% (95% of the time); if 30 games have been played, the margin of error is less than 18%; if 100 games have been played, the margin of error is less than 10% and, if 1000 games have been played, the margin of error is less than 3%.

* * * * *

Now that John understands the law of large numbers, Peter explains that for any game, for which the results can be compiled statistically, the expectation for one or a series of sessions is calculated as follows:

$$E = \frac{\text{Cumulative Gains (or losses)}}{\text{Cumulative Amount Wagered}} \qquad \text{Formula 4}$$

The single-trial probability of success may be calculated by modifying Formula 3 in the following manner:

$$P = .5 + E/2$$

$$P = .5 + \frac{\text{Cumulative Gains (or losses)}}{2 \times \text{Cumulative Amount Wagered}} \qquad \text{Formula 5}$$

Peter gives the following examples to explain how to use these formulas.

Example 9:

A government runs a lottery and, after a few years of operation, their figures indicate that the value of the ticket sales is \$500 million and that the prizes total \$300 million. What is the single-trial probability of success for the government?

The cumulative gain is \$200 million (\$500 million - \$300 million) and the cumulative amount wagered is \$500 million. The expectation is obtained using Formula 4:

$$E = \frac{\text{Cumulative Gains (or losses)}}{\text{Cumulative Amount Wagered}} = \frac{200 \text{ million}}{500 \text{ million}} = 40\%$$

The single-trial probability of success is evaluated using Formula 3:

$$P = .5 + E/2 = 50\% + 40\%/2 = 70\%$$

The single-trial probability of success for the government is 70%.

Example 10:

A casino estimates, that in the long run, it wins an average of $100 an hour at a blackjack table where the minimum bet is $2. With the assumption that the average number of players per table is 4, that there are 50 deals per hour and that each player bets an average of $5, what is the single-trial probability of success for the casino?

The amount wagered per hour is obtained as follows:

4 players x 50 deals/hour x $5/each = $1000/hr

The hourly gain is $100 and the expectation expressed in percentage is calculated using formula 4:

$$E = \frac{\text{Cumulative Gains (or losses)}}{\text{Cumulative Amount Wagered}} = \frac{\$100}{\$1000} = 10.0\%$$

The single-trial probability of success is evaluated using Formula 3:

$$P = .5 + E/2 = 50\% + 10.0\%/2 = 55.0\%$$

Therefore, based on the above assumptions, the single-trial probability of success for the casino is 55.0%.

Example 11:

A backgammon player has compiled the following results:

Game number	Result of each game	Cumulative result
1	+1	+1
2	-2	-1
3	+4	+3
4	-1	+2
5	+2	+4
6	-1	+3
7	+4	+7
8	-2	+5
9	+1	+6
10	-2	+4

The player plays at $5 a point. What is the amount won? What is the cumulative amount wagered? What is the single-trial probability of success? What is the margin of error of this result?

The player won four points or $20. To evaluate the cumulative amount wagered, it is necessary to calculate the total number of points, and to do this, it is suggested to proceed as follows:

Number of games ended at level 1 (4 games x 1 point):	4 points
Number of games ended at level 2 (4 games x 2 points):	8 points
Number of games ended at level 4 (2 game x 4 points):	8 points
Total:	20 points

There was a total of 20 points (whether won or lost), and at $5 a point this corresponds to the cumulative amount wagered of $100. The single-trial probability of success is established with Formula 5:

$$P = .5 + \frac{\text{Cumulative Gains (or losses)}}{2 \times \text{Cumulative Amount Wagered}} = .5 + \frac{\$20}{2 \times \$100} = 60.0\%$$

The single-trial probability of success is 60.0%. In other words, the player has won 60% (or $60) of the cumulative amount wagered ($100); he has lost 40% (or $40) of the cumulative amount wagered, and he has an expectation of 20%. To have an expectation of 20% corresponds to having a single-trial probability of success of 60%. Since only 10 games have been played, the results obtained have a margin of error of less than 31% (95% of the time). In other words, the true probability of winning can also be 40% or 65%. Based on these results, a player cannot justify his superiority.

* * * * *

Peter explains that, if a backgammon player wants to know the single-trial probability of success against an opponent, he must compile the results obtained from each session. The method of compiling these results is indicated below.

Table 2
The table to use for calculating the single-trial probability of success for backgammon

Date	Number of games	Gains or losses ($)	Total amount wagered ($)	Cumulative number of games	Cumulative gains or losses ($)	Cumulative amount wagered ($)	P (%)

To keep this table up-to-date takes less than five minutes each session. Peter gives John the following example to show how such a table can be kept up-to-date.

Example 12:

Dick always plays backgammon against the same opponent. The results obtained from each session are the following:

Session no 1: 100 games were played at $5 a point. Dick won $100 and the total number of points (which could have been won or lost) was 200. (The total amount wagered was $1000).

Session no 2: 50 games were played at $5 a point. Dick lost $20 and the total number of points was 90. (The total amount wagered was $450). For sessions 1 and 2, the cumulative number of games is 150, the cumulative gain is $80 and the cumulative amount wagered is $1450.

Session no 3: 100 games were played at $10 a point. Dick won $50 and the total number of points was 170. (The total amount wagered was $1700).

Session no 4: 60 games were played at $5 and 40 games at $10. Dick lost a total of $50. The total amount wagered was $1100.

Session no 5: 5 matches at $50 a match were played, Dick won 3 of them. This is the equivalent of playing 5 games at $50 a game. Dick won $50 and the total amount wagered was $250.

Session no 6: Dick did not keep any records, but he won 10 points at $10 each. The duration of the session was about 5 hours and they played around 12 games an hour, for an approximate total of 60 games. From previous sessions, he knows that his cube factor is 2.0 (refer to section 6.3 for explanations of the cube factor). Therefore, Dick guessed that the total amount wagered was $1200 (60 games x Cube factor of 2.0 x $10 a point).

From the above information, Peter asks John: "Can you calculate the single-trial probability of success after each session?".

Completing Table 2, John gets the following results:

Date	Number of games	Gains or losses ($)	Total amount wagered ($)	Cumulative number of games	Cumulative gains or losses ($)	Cumulative amount wagered ($)	P (%)
1	100	100	1000	100	100	1000	55.0
2	50	-20	450	150	80	1450	52.8
3	100	50	1700	250	130	3150	52.1
4	100	-50	1100	350	80	4250	50.9
5	5	50	250	355	130	4500	51.4
6	60	100	1200	415	230	5700	52.0

After 415 trials (games and matches), Dick has won $230, the cumulative amount wagered is $5700 and the single-trial probability of success is evaluated at 52.0%. After 415 trials, this percentage has a margin of error of less than 5% (95% of the time).

* * * * *

The previous example takes into account all the games played in six sessions. To establish the single-trial probability of success, using all previously recorded information is not necessarily the best approach. For example, records which are older than one year can be misleading. If an opponent's calibre of play has improved, it may be more accurate to use the results of only the last 6 months or the last 1000 games. The most important thing is, first, to keep records of your opponents, and secondly, to correctly assess them by using only significant data. Notwithstanding the shortcomings mentioned above, Peter explains that the single-trial probability of success is always calculated in the same way as in the preceding example.

Now that John is able to calculate the expectation, the hourly expectation and the single-trial probability of success for any game including backgammon, he asks Peter to go on to the next concept.

CHAPTER 2 - CALCULATION OF THE PROBABILITY OF SUCCESS

To understand the principles of money management, Peter explains that it is important to understand the concept of the probability of success. He will give the formulas to be used and describe how to apply them.

Peter explains that a player who can determine the three following variables:

P = single-trial probability of success,

X = amount of money risked (X dollars),

Y = amount of money "to win" (Y dollars),

can calculate **his probability of success, which is to say, the probability of winning Y dollars instead of losing X dollars.** This probability depends on "P", the single-trial probability of success and on the importance of the unit bet. The formulas used for calculating the probability of success are:

if P = 50%, $P_{success} = \frac{A}{A+B}$ Formula 6

if P ≠ 50%, $P_{success} = \frac{(Q/P)^{A} - 1}{(Q/P)^{A+B} - 1}$ Formula 7

P = single-trial probability of success

Q = single-trial probability of ruin (Q = 1-P)

$P_{success}$ = probability that the player wins B units (instead of losing A units)

A = Number of units risked = $\frac{\text{Amount risked}}{\text{Unit bet}}$

B = Number of units that the player desires to win = $\frac{\text{Amount to win}}{\text{Unit Bet}}$

(The formulas 6 and 7 are from the book "The Theory of Gambling and Statistical Logic", page 59). (The symbol ≠ means "not equal to".)

John is a bit frustrated because these formulas mean nothing to him. Peter, on the other hand, maintains that, as long as John can grasp the examples, he will have learnt the most important thing, i.e. the meaning of these formulas.

To be able to utilize these formulas, John has to establish, as objectively as possible, the following variables: 1) the single-trial probability of success; 2) amount to risk; 3) amount to win and 4) amount of the unit bet.

1) The single-trial probability of success (See Chapter 1)

2) Amount to risk

First of all, the amount of money to risk depends on subjective and personal criteria. That amount is therefore, not always easy to calculate with any kind of precision. The amount to use in Formulas 6 or 7 has to be the amount that the player is willing to risk for the session he is involved in. Peter supports this idea with the following examples.

Example 13:

A blackjack player goes to Las Vegas with $1000 for expenses and $3000 for gambling, hoping to win the latter amount. Of the money to be risked, what amount has to be used in Formulas 6 or 7?

The amount to be utilized is the amount the player is willing to risk while playing in each session. If the player intends to risk a maximum of $500 in the first session and if he intends to leave the game upon losing this amount, then the amount to be risked is $500. The probability of success thus obtained in using $500 (if that is the case) will be valid for the first session. If, on the other hand, the player is willing to risk his $3000 up to the point of reaching his goal or losing his $3000, then the latter amount must be used as the amount to be risked.

Example 14:

Phil, another backgammon player, has put aside in a special bank account his winnings from backgammon; these gains represent $2000. Phil has $400 on him. What is the amount of money at risk?

To establish the amount to be used in Formulas 6 or 7, Phil must utilize the entire amount of money that he is willing to risk while playing. The answer is therefore $2000, if Phil wants to risk all of his past gains, $400, if he wants to risk all the money he has on him, and $200 if, for example, Phil wants to risk 10% of his past gains.

3) Amount to win

A remark must be made about determining the amount to win. This notion is very subjective, depending on the opponent. A player who wants to win $10,000 in a casino in Las Vegas can be sure he will have no problem being paid if he wins. On the other hand, it may be unrealistic for a backgammon player to aim at a goal of $10,000 when facing an unknown opponent. Practically speaking, the amount of money to win must often be established taking into account what your opponent will pay if he loses. It is necessary to insist that Formulas 6 and 7 be established with the hypothesis that the "money is on the table", or that payment is assured.

4) Amount of the unit bet

The results given by Formulas 6 and 7 are exact if, and only if, the bet is uniform and constant. This does not mean that the player has to play with a uniform bet, but if he doesn't play with a uniform bet, he cannot use Formulas 6 and 7. By using an average bet nevertheless, the result obtained may be acceptable.

* * * * *

It must be pointed out that Formulas 6 and 7 give exact results subject to the exactness of the given entries. For example, John determines the following: 1) P = Single-trial probability of success = X%, 2) Money to risk = $200, 3) Money to win = $200, 4) Unit bet (uniform) = $10; and calculates $P_{success}$ according to Formulas 6 or 7. If he doesn't quit, when he has already won the amount he decided upon previously, or when he has lost the amount he was willing to risk, then the probability of success that he had initially calculated is no longer valid. On the other hand, if the given entries are objectives and if the player quits when he wins the determined amount, or loses the amount that he had to risk, it follows that the probability of success calculated from Formulas 6 and 7 is exact. Peter gives the following examples to explain how to calculate the probability of success.

Example 15:

Dick wants to play a game in which the single-trial probability of success is 50% (for example, heads or tails). He has \$100 to risk to win \$10, and will play with a uniform bet of \$10 until the moment he either wins \$10 or loses \$100. What is the probability that Dick will win \$10?

Since the single-trial probability of success is 50%, Formula 6 can be applied:

$$P_{success} = \frac{A}{A + B}$$

The unit is \$10, Dick has \$100 or 10 units to risk and A = 10. Dick has \$10 to win, the equivalent of one unit, therefore B = 1.

$$P_{success} = \frac{A}{A + B} = \frac{10}{10 + 1} = \frac{10}{11} \approx 91\%$$

(The symbol $\approx$ means "approximately equal to".) The probability of success for Dick is therefore 91%. The fact that Dick has a probability of success of 91% doesn't mean that he has a positive expectation. Actually, his expectation is nil because, if Dick plays 110 times the following proposition "I have \$100 to risk to win \$10. I play until the moment I either win \$10 or lose \$100", he should win 100 times \$10 (winning \$1000) and he should lose 10 times \$100 (losing \$1000). Therefore, if the single-trial probability of success is 50%, the expectation is nil even if the probability of success is 91% (if P = 50% therefore E = 0). Peter points out to John that many gamblers believe that a player, having more money than his opponent, has thereby a positive expectation. This example clearly indicates that this belief is unfounded.

Example 16:

Dick plays a game in which P = 50%. He has \$100 to risk to win \$100. If the unit bet is \$10, what is $P_{success}$?

Since P = 50%, it is necessary to use Formula 6. The unit is \$10, Dick risks 10 units (A = 10) to win 10 units (B = 10). The probability of success is:

$$P_{success} = \frac{A}{A + B} = \frac{10}{10 + 10} = \frac{10}{20} = 50\%$$

Peter points out that if the unit bet had been \$1, A would have been equal to 100 (A = 100) and B would have been equal to 100 (B = 100), and the probability of success would have been unchanged. Therefore, when P = 50%, the probability of success does not depend on the amount of the unit bet.

Example 17:

Dick plays a game of Chuck-A-Luck in a casino. His single-trial probability of success is 46.1% (see example 6). He has $100 to risk to win $10. If the unit bet is $10, what is $P_{success}$?

Since P $\neq$ 50%, it is necessary to use Formula 7:

$$P_{success} = \frac{(Q/P)^A - 1}{(Q/P)^{A+B} - 1}$$

If P = 46.1%, then Q = 53.9% and Q/P = 53.9%/46.1% = 1.16920. The unit bet is $10, Dick has $100 to risk, which is to say A = 10 units. He wants to win $10 or one unit, therefore B = 1. We have as follows:

$$P_{success} = \frac{(1.16920)^{10} - 1}{(1.16920)^{10+1} - 1} = .8237 \approx 82\%$$

Dick has a probability of success of 82%. Peter notices that John does not understand this example due to his lack of experience with pocket calculators. He takes five minutes, therefore, to illustrate how a similar problem could be calculated:

1) He asks John to calculate $\frac{4+6}{2}$. John presses 4 + 6 ÷ 2 and obtains 7 while the correct answer is 5. Although certain calculators give an answer of 5, Peter explains that with others, after having punched "4 + 6", he must press on the "=" before dividing by 2. John has to "understand" the calculator he is working with, and the best method is by doing simple calculations.

2) He asks John to calculate $\frac{4+6}{2+3}$. After entering the amounts, John does not arrive at the correct answer of 2.0. Peter explains that he must execute it in the following sequence: (4 + 6) ÷ (2 + 3) = .

3) Peter indicates to John that 1.2^{10} equals 6.19 and that $1.2^{10} - 1$ equals 5.19. John makes this calculation easily.

4) Peter points out that: $\frac{1.2^{10} - 1}{1.2^{11} - 1} = \frac{5.19}{6.43} = .807$ or 81% and that this calculation can be made easily. John is surprised to find that he can complete these calculations in less than 30 seconds.

With these explanations, John is now able to check the calculations involved in this example and arrives at the correct answer, which is 82%.

Example 18:

Dick, who plays Chuck-A-Luck ($P = 46.1\%$), has \$100 to risk to win \$10. If the unit bet is \$1, what is $P_{success}$?

If $P = 46.1\%$, then $Q/P = 1.16920$. Since the unit is \$1, Dick is risking 100 units ($A = 100$) to win 10 units ($B = 10$). The probability of success is calculated from Formula 7:

$$P_{success} = \frac{(1.16920)^{100} - 1}{(1.16920)^{100+10} - 1} = .2095 \approx 21\%$$

John points out to Peter that he finds the above theoretical result of 21% to be a very weak proportion considering the amounts involved (\$100 risked to win only \$10). Peter answers that he is convinced that the theoretical result of 21% is exact and immediately suggests the following proposition:

> "A player who plays Chuck-a-Luck has \$100 to risk to win \$10. This player plays with a uniform bet of \$1. If he wins \$10 before losing \$100, it is a success. If on 100 trials this player has more than 30 successes, the I lose X dollars; if not, I win X dollars".

Peter points out that, in accordance with the law of large numbers (as elaborated in Table 1 and Diagram 1, when $P = 50\%$), with 100 trials, the number of successes obtained 95% of the time will be 21% with a margin of 10%, i.e. between 11 and 31 successes. (A good mathematician will notice that a more exact approach, for the case where $P = 21\%$, gives between 13 and 29 successes, 95% of the time.)

John is still perplexed. On the one hand, he doubts the theoretical result of 21%, and, on the other, he is not ready to accept Peter's proposition. Peter points out that his goal is not to give a theoretical demonstration of how Formula 7 is obtained and that the only practical way to check the theoretical results obtained is through a large number of trials.

* * * * *

John believes that the concept of probability of success is difficult to apply because it is difficult to know the exact moment that the opponent will quit. To show Peter what he means, John offers the next example.

Example 19:

Dick plays a game where P = 50%. Dick has $100 to risk to win $10; as in Formula 6, he has a probability of success of 91%. Dick plays against Phil who has a goal of winning $10. If Phil wins $10 and then quits, Dick has therefore won nothing. Based on these considerations, John concludes that, since the probability of success depends on the moment when the opponent quits, it follows that this concept is an invalid one. Is this conclusion correct?

Peter points out that the concept of the probability of success is a concept which is completely independent of the number of games to be played. If Dick plays until he either wins what he has to win or loses what he has to lose, whatever number of games this takes, (10, 100, 1000 games or more), then the probability of success established with Formulas 6 and 7 is exact. The fact that Phil quits before Dick has lost what he has to risk or won what he has to win, doesn't mean that the probability of success is invalid.

It is necessary to make a distinction between the concept of "making money in one session against an opponent" and the concept of the "probability of success." Again, Peter states that this last concept has, as an essential condition, the obligation for Dick to play until he wins what he has to win or loses what he has to lose. Peter concludes that the concept of the probability of success is subject neither to the number of games to be played, nor to the number of opponents to be played against.

CHAPTER 3 - CALCULATION OF THE PROBABLE NUMBER OF GAMES

The concept of the probable number of games to be played is an important one because it permits an estimation of the time required to obtain the desired success (or ruin). In other words, this concept enables one to evaluate the time required to win B units or lose A units.

The formulas to utilize for calculating the probable number of games are:

if P = 50%, $N_{prob.} = A \times B$ Formula 8

if P ≠ 50%, $N_{prob.} = P_{success} \times \frac{(A+B)}{(P-Q)} - \frac{A}{(P-Q)}$ Formula 9

P	= single-trial probability of success
Q	= single-trial probability of ruin ($Q = 1-P$)
$N_{prob.}$	= probable number of games
A	= number of units risked
B	= number of units that the player desires to win
$P_{success}$	= probability of success like that calculated with Formula 7

(The Formulas 8 and 9 come from "The Theory of Gambling and Statistical Logic", page 66).

Example 20:

Dick plays a game where P = 50%, he has 10 units to risk (A = 10) to win 10 units (B = 10). What is the probable number of games to be played? At 10 games per minute, what is the probable time?

Since P = 50%, we use Formula 8:

$$N_{prob.} = A \times B = 10 \times 10 = 100 \text{ games}$$

Therefore, the probable number of games is 100 and the probable time is 10 minutes.

Example 21:

Dick who plays a game of Chuck-A-Luck (P = 46.1%), has $100 to risk to win $100. If the unit bet is $10, what are $P_{success}$ and N_{prob}? At 10 games per minute, what is the probable time?

Since P = 46.1%, it is necessary to use Formulas 7 and 9. If P = 46.1%, then Q = 53.9% and Q/P = 1.16920. The unit is $10, the player therefore risks 10 units (A = 10) to win 10 units (B = 10).

$P_{success}$ and N_{prob} are calculated as follows:

$$P_{success} = \frac{(Q/P)^A - 1}{(Q/P)^{A+B} - 1} = \frac{(1.16920)^{10} - 1}{(1.16920)^{10+10} - 1} = .1732$$

$$N_{prob.} = P_{success} \times \frac{(A+B)}{(P-Q)} - \frac{A}{(P-Q)}$$

$$N_{prob.} = .1732 \times \frac{(10+10)}{(.461-.539)} - \frac{10}{(.461-.539)}$$

$$N_{prob.} = -44 + 128 = 84 \text{ games}$$

Therefore, $P_{success}$ = 17.32%, N_{prob} = 84 games, and the probable time for Dick to win $100 or lose $100 is evaluated at 8 minutes.

* * * * *

Now John indicates to Peter that he knows:

- how to calculate the expectation of a game (E);
- how to calculate the hourly expectation;
- how to calculate the single-trial probability of success (P);
- how to use the law of large numbers;
- how to calculate the probability of success ($P_{success}$);
- how to calculate the probable number of games (N_{prob});

but he would like to know if he really has to use all these concepts to be able to establish an appropriate bet.

Peter points out to John that:

a) casinos have been built with the formula of probability of success; roughly speaking, the average advantage of a casino (in general) varies from 2% to 10%. In other words, their single-trial probability of success varies from 51% to 55%;

b) an expert backgammon player has approximately 5% to 20 % advantage over an intermediate player. In other words, his single-trial probability of success varies from 52.5% to 60%;

c) if a player understands the above concept, then it will be easier for him to grasp the real importance and meaning of the basic principles of money management.

CHAPTER 4 - THE SUGGESTED CRITERIA FOR DETERMINING THE APPROPRIATE BET FOR GAMES IN GENERAL

Peter explains that, using the concepts outlined in the preceding sections, it is possible to establish the appropriate bet for games in general. The four following criteria will be reviewed:

1) the maximization of the probability of success;
2) the maximization of the hourly expectation;
3) a pre-established probability of success (for favorable games);
4) a pre-established number of games.

Peter notes that it is also possible to maximize the rate of increase of wealth by using the Kelly system (if the game is favorable). This last criterion is very interesting, but it will not be analyzed because it is not useful for backgammon; Peter directs John to the following references:

- Epstein, Richard A., "The Theory of Gambling and Statistical Logic (1977)", Academic Press Inc., pages 60, 61 and 62.

- Wilson, Allan N., "The Casino Gambler's Guide", Harper & Row Publishers (Enlarged Edition), pages 298, 299 and 300.

- Thorp, Edward O., "The Kelly Money Management System", Gambling Times Magazine, December 1979, pages 91-92.

4.1 Criterion No. 1: Determination of the bet in terms of the maximization of the probability of success

John knows that with the following variables:

P = single-trial probability of success

A = number of units that a player is ready to risk

B = number of units that a player desires to win

it is possible to calculate:

$P_{success}$ = probability that a player who risks A units, wins B units.

To determine the bet in terms of the maximization of the probability of success, Peter points out that there are three possibilities to be considered:

A) **The game is favorable (P > 50%)**

For favorable games with a single-trial probability of success of over 50% (P > 50%), the probability of success is calculated using Formula 7. Peter presents, in Table 3, a practical illustration of the variation of the probability of success in terms of the unit bet.

Table 3

The determination of the probability of success in terms of the value of the unit bet when P = 55%, money risked = $100, money to win = $100

Value of the unit bet	Probability of success (approx.)
$100	55%
$ 50	60%
$ 20	73%
$ 10	88%
$ 5	98%
$ 2	100%

Table 4 makes it evident that, in favorable games, the probability of success increases when the value of the unit bet decreases.

B) **The game is fair (P = 50%)**

Peter stresses that for games where the single-trial probability of success is equal to 50% (P = 50%), the probability of success can be calculated using Formula 6. The probability of success doesn't depend on the value of the unit bet, but on the amount risked in terms of the amount to be won. If the amount risked is equal to the amount to be won, it follows that the probability of success is 50%. John agrees on this point.

C) **The game is unfavorable (P < 50%)**

For unfavorable games, which is to say, when the single-trial probability of success is less that 50% (P < 50%), the probability of success is calculated using Formula 7. In Table 4, Peter presents a practical illustration of the variation of the probability of success in terms of the unit bet.

Table 4

The determination of the probability of success in terms of the value of the unit bet when P = 45%, money risked = $100, money to win = $100

Value of the unit bet	Probability of success (approx.)
$100	45%
$ 50	40%
$ 20	27%
$ 10	12%
$ 5	2%
$ 2	0%

Table 5 illustrates that, for games where P is less than 50%, the probability of winning increases when the unit bet increases, but that the probability of success is always less than 50%.

Peter presents, in Diagram 2, the results obtained so far.

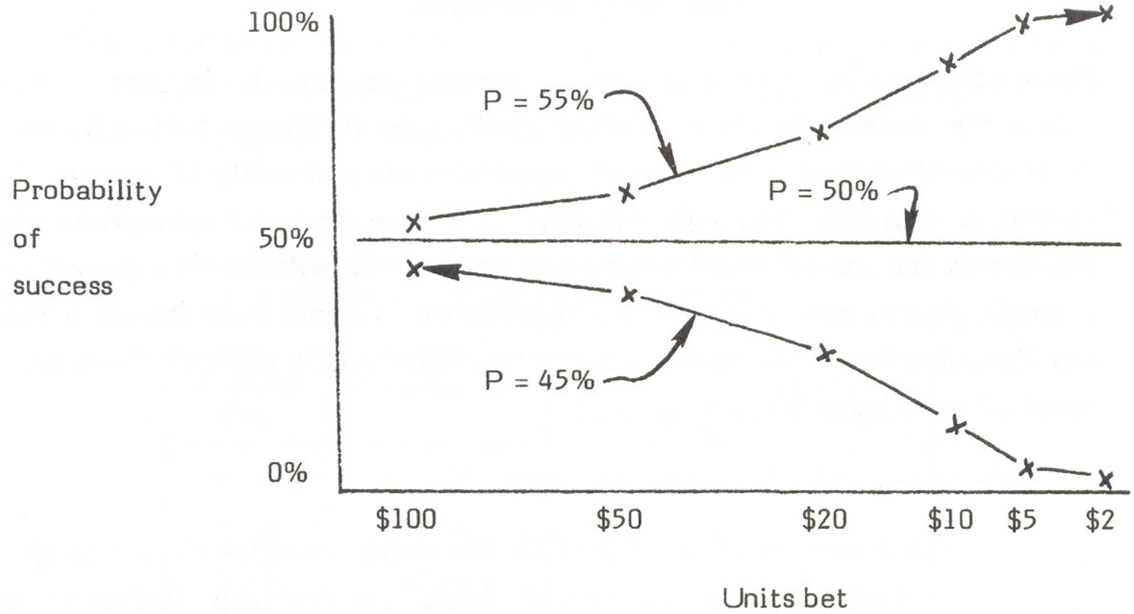


Diagram 2: Probability of success
P = 45%, 50% and 55%
Money risked = $100
Money "to win" = $100

Peter and John conclude that, for games in general (including backgammon), the player who wants to determine which bet will maximize his probability of success must follow these principles:

1) if the game is favorable ($P > 50\%$), the player should play with the smallest bet possible;

2) if the game is fair ($P = 50\%$), the bet is not important; and

3) if the game is unfavorable ($P < 50\%$), the player should play with the largest bet possible, but in this last case, it must be remembered, $P_{success}$ is always less than 50%.

4.2 Criterion No. 2: Determination of the bet in terms of the maximization of the hourly expectation

Peter explains a second criterion that a gambler may use. If the time necessary to obtain the desired objective is unimportant, then it follows that a player must determine his bets only in terms of maximizing his probability of success. Nevertheless, as a general rule, **time is money.** Therefore, the most appropriate bet isn't necessarily the one which maximizes the probability of success. The player may, for example, play to maximize his hourly expectation. To determine the bet in terms of the maximization of the hourly expectation, Peter points out that there are three possibilities to consider:

A) The game is favorable (P > 50%)

For favorable games (P > 50%), the hourly expectation is obtained from Formula 2. Peter presents, in Table 5, a practical illustration of the variation of the hourly expectation in terms of the units bet.

Table 5

The calculation of the hourly expectation in terms of the value of the average bet when P = 55% (or E = 10%) 10 games per hour

Average bet	Expectation	Games per hour	Hourly expectation
$ 100	10%	10	$ 100
$ 50	10%	10	$ 50
$ 20	10%	10	$ 20
$ 10	10%	10	$ 10
$ 5	10%	10	$ 5
$ 2	10%	10	$ 2

Even though Peter finds that this table is an obvious application of Formula 2 (hourly expectation = average bet x expectation x games per hour) and appears superfluous, he remarks that it makes the matter easier to understand.

B) The game is fair (P = 50%)

For games in which the single-trial probability of success is equal to 50% (P = 50%), the expectation is nil and the hourly expectation is also nil.

C) The game is unfavorable (P < 50%)

For unfavorable games (P < 50%), the hourly expectation is obtained from Formula 2. Peter presents a practical illustration of the variation of the hourly expectation in terms of the units bet in Table 6.

Table 6
The calculation of the hourly expectation in terms of the value of the average bet when P = 45% (or E = -10%) 10 games per hour

Average bet	Expectation	Play per hour	Hourly expectation
$ 100	-10%	10	- $ 100
$ 50	-10%	10	- $ 50
$ 20	-10%	10	- $ 20
$ 10	-10%	10	- $ 10
$ 5	-10%	10	- $ 5
$ 2	-10%	10	- $ 2

Peter presents, in Diagram 3, the results obtained until now.

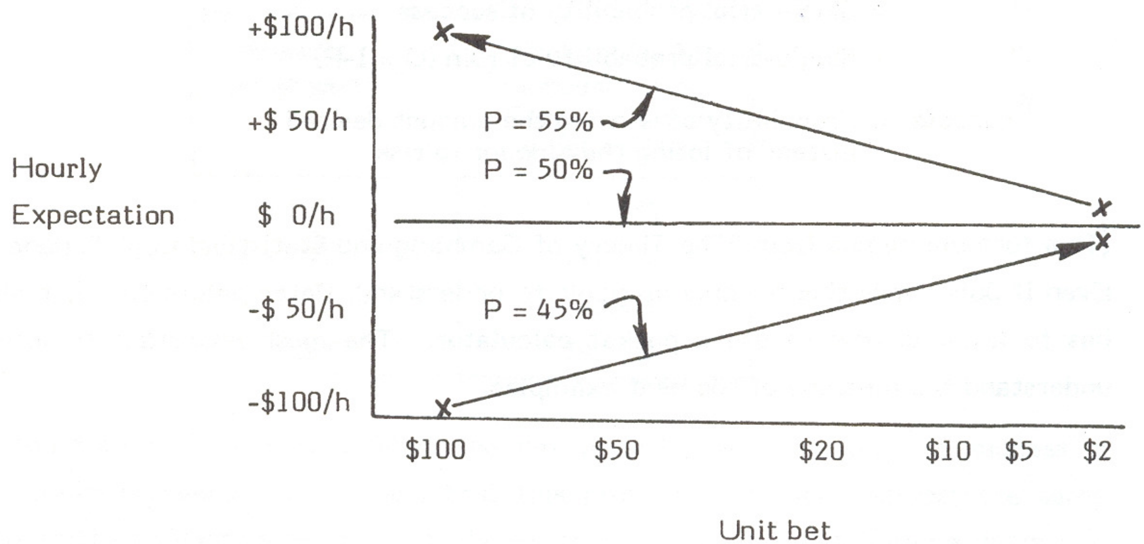


Diagram 3: Hourly Expectation
P = 45%, 50%, 55%
10 games per hour

Peter and John conclude that, for games in general (including backgammon), the player who wants to determine his bet in order to maximize his hourly expectation must follow these principles:

1) if the game is favorable ($P > 50\%$), the player should play with the largest bet possible;
2) if the game is fair ($P = 50\%$), the bet is not important and the expectation is nil; and
3) if the game is unfavorable ($P < 50\%$), the player should play with the smallest bet possible.

4.3 Criterion No. 3: Determination of the bet in terms of a pre-established probability of success (for favorable games)

This is a third criterion that a gambler can use. For favorable games, a player may use a pre-established probability of success to determine the bet. When the amount desired to be won is equal or superior to the amount risked, the unit bet may be approximated as follows:

$$\text{Appropriate bet} \approx \text{Amount risked} \times \frac{\text{Log}(Q/P)}{\text{Log}(1-P_{\text{success}})} \qquad \text{Formula 10}$$

P = Single-trial probability of success

Q = Single-trial probability of ruin ($Q = 1-P$)

P_{success} = Probability of winning the amount desired instead of losing the amount to risk

(This formula comes from "The Theory of Gambling and Statistical Logic", page 62). Even if John finds this formula difficult to understand, Peter points out that all he has to know is how to use a pocket calculator. The most important thing is to understand the meaning of the next examples.

Example 22:

Dick asks Phil to play Chuck-A-Luck. Dick has a single-trial probability of success equal to 53.9% (see example 6). Dick has $100 to risk and Phil has an equal or larger amount to risk. If Dick wants to obtain a probability of success of about 80%, what is the appropriate bet? What will be the accuracy of the result obtained?

To obtain a probability of success of about 80%, Peter uses Formula 10 as follows:

$$\text{Appropriate bet} \approx \text{Amount risked} \times \frac{\text{Log}(Q/P)}{\text{Log}(1-P_{\text{success}})}$$

$$\text{Appropriate bet} \approx \$100 \times \frac{\text{Log}(46.1\%/53.9\%)}{\text{Log}(1-.80)} \approx \$9.71 \approx \$10$$

To have a pre-established probability of success of about 80%, therefore, the appropriate bet is $10. Peter points out that the only way to check the accuracy of the above result is to use Formula 7 seen previously and change the amount to be won to find out the probability of success. The following table gives the exact probability of success in relation to the amount to be won, when the amount risked is $100:

Amount to be won	Probability of success (using Formula 7 with a bet of $10)
$ 100	82.68%
$ 200	79.79%
$ 500	79.06%
$ 1 000	79.05%

This table demonstrates that when the bet is $10, the probability of success is approximatively equal to 80% and that this probability remains almost the same whatever the amount to be won. A player, then, involved in a favorable game and wishing to obtain a pre-established probability of success, can find his appropriate bet by using Formula 10.

4.4 Criterion No. 4: Determination of the bet in terms of a pre-established number of games

Peter will explain this fourth and last criterion. He stresses that it is possible for a player to establish his own betting level to play for 100, 500 or 1000 games, or, to play for a certain number of hours, be it 1 hour, 5 or 10 hours. As an approximation, it is possible to use Formula 8 which is valid when P = 50% and assume that the amount of money to win is equal to the amount of money risked. Formula 8 is:

If P = 50%, $N_{prob} = A \times B$

$N_{prob.}$ = probable number of games

$$A = \text{number of units risked} = \frac{\text{Amount risked}}{\text{Unit bet}}$$

$$B = \text{number of units that the player desires to win} = \frac{\text{Amount to win}}{\text{Unit bet}}$$

On the basis of the above assumption, Peter makes the following substitutions:

$$B = \frac{\text{Amount to win}}{\text{Unit bet}} = \frac{\text{Amount risked}}{\text{Unit bet}}$$

$$N_{prob.} = A \times B = \frac{(\text{Amount risked})^2}{(\text{Unit bet})^2}$$

Isolating the "Unit bet", Peter has the following formula:

$$\text{Unit bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob.}}} \qquad \text{Formula 11}$$

N_{prob} = Number of probable games.

Example 23:

Dick plays a game where the single-trial probability of success is about 50%. He has \$100 to risk and wants to play about 100 games. What is the appropriate bet to allow him to attain this goal?

Using Formula 11, John obtains:

$$\text{Unit bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob.}}} = \frac{\$100}{\sqrt{100}} = \$10$$

Therefore, if Dick plays at \$10 a game, he will probably play 100 games.

CHAPTER 5 - COMPLETE STRATEGY FOR GAMES IN GENERAL

Before going further, Peter asks John to make a clear summary of the principles discussed so far. John prepares the following summary:

1) The player who knows:

   P = single-trial probability of success

   A = number of units risked = Amount risked/Unit bet

   B = number of units to win = Amount to win/Unit bet

   can calculate:

   a) $P_{success}$ which is the probability of winning the desired amount opposed to losing the amount risked; to do this, it is necessary to use Formulas 6 and 7.

   b) N_{prob} which is the probable number of games to play; to do this, it is necessary to use Formulas 8 and 9.

2) If the player wishes to maximize his probability of success, he should, in principle, play with the minimum bet if the game is favorable ($P > 50\%$) and he should play with the maximum bet if the game is unfavorable ($P < 50\%$).

3) If the player wants to maximize his hourly expectation, he should, in principle, play with the maximum bet if the game is favorable ($P > 50\%$) and play with the minimum bet if the game is unfavorable ($P < 50\%$).

4) If the player's goal is to play in terms of a pre-established probability of success and if the game is favorable ($P > 50\%$), then the appropriate unit bet can be approximated using Formula 10.

5) If the player wishes to play a pre-established number of games, he can approximate the unit bet using Formula 11.

Peter congratulates John and stresses that these formulas and principles can guide and aid players in determining their appropriate bet.

Noting that four criteria have been analyzed and that each criterion gives different results, it can be seen that the notion of the appropriate bet is entirely subjective. What is convenient for one player may not be so for another.

The formulas used to calculate $P_{success}$ (probability of winning B units opposed to losing A units) and N_{prob} (number of probable games to play) come from "The Theory of Gambling and Statistical Logic" and, more specifically, from the chapter entitled "Fundamental Principles of a Theory of Gambling". This chapter, which analyzes "betting systems" in a general fashion, gives the following conclusion:

> "A definitive theory depends, first and foremost upon utility goals - that is, upon certain restricted forms of subjective preference. Within the model of strictly objective goals, we have shown that no betting system can alter the mathematical expectation of a game; however, probability of ruin (or success) and expected duration of a game are functions of the betting system employed." (page 72)

This book concludes that "no betting system can alter the mathematical expectation of a game". Peter asks John if he really understands the significance of such a conclusion. The fact that a player plays \$1, \$2, \$5, \$10, \$20, \$50 or \$100 a game doesn't in theory change the mathematical expectation. Whatever the betting system used (Martingale, etc.), the mathematical expectation will not change, and **an unfavorable game will always remain an unfavorable game.** The first reflex of a player should be to evaluate, as impartially as possible, his single-trial probability of success. The determination of P is of great importance, since all the formulas analyzed up to now depend upon it.

Once the player has evaluated whether a game is favorable ($P > 50\%$), fair ($P = 50\%$) or unfavorable ($P < 50\%$), the next step is to decide which criterion to use. Peter explains that there are three cases to consider:

A) The game is favorable (P > 50%)

The appropriate bet for a player who wants to maximize his probability of success is the minimum bet. If the player wants to maximize his hourly expectation, then the appropriate bet is the maximum bet. If a player wants to play with a pre-established probability of success, then he should use Formula 10, namely:

$$\text{Appropriate bet} \approx \text{Amount risked} \times \frac{\text{Log}(Q/P)}{\text{Log}(1-P_{success})}$$

And finally, if a player wants to play a pre-established number of game, he should use Formula 11, namely:

$$\text{Unit bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob.}}}$$

B) The game is unfavorable (P < 50%)

The appropriate bet for a player who wants to maximize his probability of success is the maximum bet. If the player wants to maximize his hourly expectation, the appropriate bet is the minimum bet. And finally, if a player wants to play a pre-established number of games, he should use Formula 11.

C) The game is fair (P = 50%)

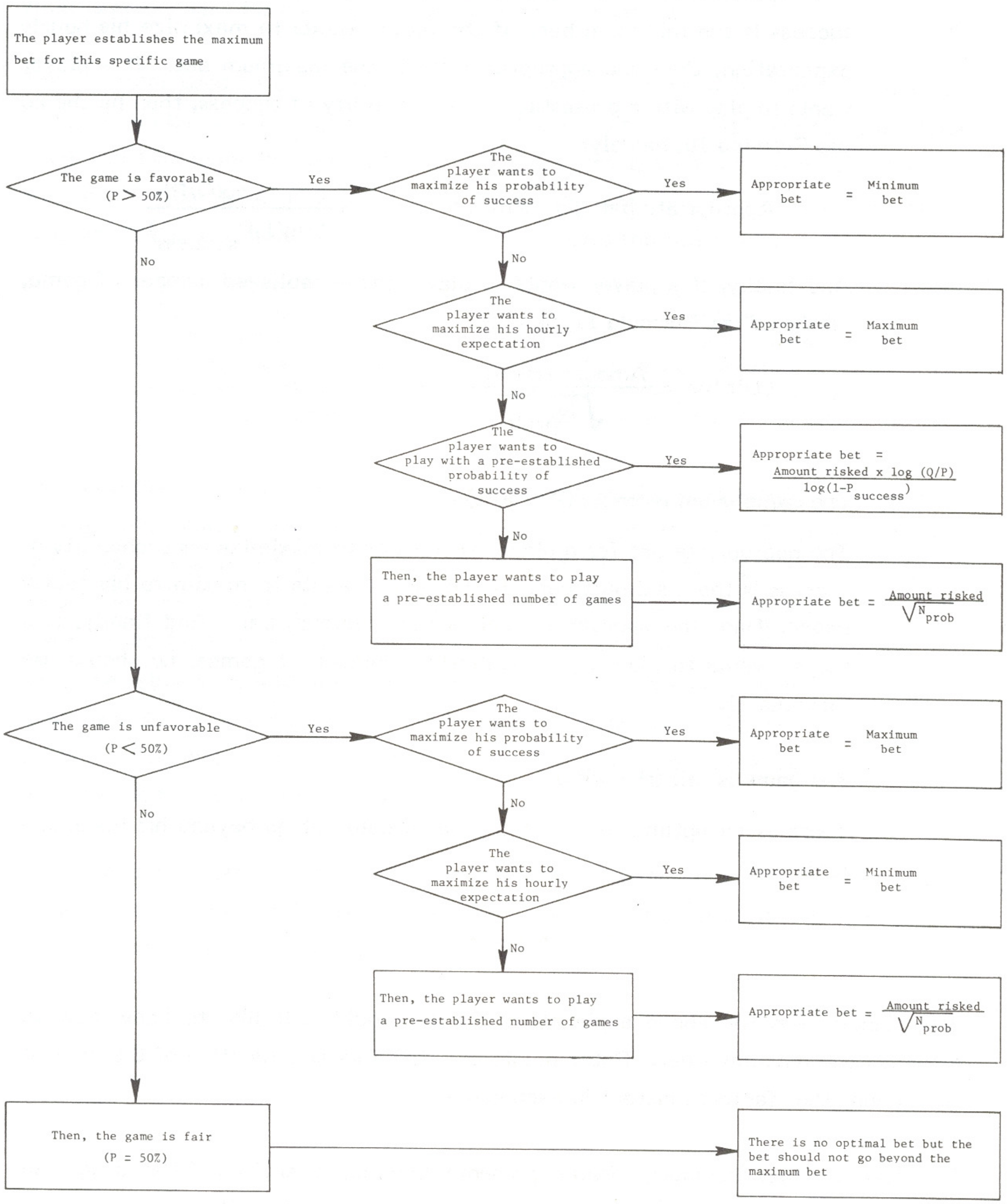
There is no optimal bet, but the bet should not go beyond his maximum limit.

* * * * *

John understands the above explanation, but he would still like to know how to establish the maximum bet. Peter points out that this is a function of the amount risked and other factors inherent in each game.

To offer a clear summary, Peter presents Diagram 4 entitled "The suggested approach for determining the appropriate bet for games in general".

Diagram 4: Suggested approach for determining the appropriate bet for games in general



Example 24:

Dick enters a casino and plays American roulette. He has $500 to risk. The minimum bet allowed is $1 and it is also possible to bet $500 at one time. If he wants to win $1000, what is the appropriate bet to obtain that goal? If Dick wants to play the longest time possible, what is the appropriate bet to obtain that goal? If he plays with a bet of $5 or $10, does he maximize something?

Referring to the Diagram 4, Dick should begin by establishing his maximum bet. In this case, the maximum is $500. Since roulette is a game that is unfavorable for the player ($P < 50\%$), the appropriate bet is the maximum bet if the player wishes to maximize his probability of success. Therefore, if Dick's goal is to maximize his probability of winning $1000, he must play with his maximum bet which is $500.

If Dick wants to play the longest time possible, then he desires to minimize his hourly losses (or maximize his hourly expectation); to accomplish this, Dick should play with the minimum bet which is $1.

Peter points out to John that if he plays an unfavorable game, then he should either play with his maximum bet of $500 to maximize his probability of success or play with his minimum bet of $1 to maximize his hourly expectation. The player who bets between the maximum and minimum maximizes neither his probability of success nor his hourly expectation. This advice is valid for all games, including backgammon.

CHAPTER 6 - SUGGESTED ADJUSTMENTS FOR BACKGAMMON

Now that John knows the principles of money management to be applied to games in general, Peter points out that, when playing a game (as opposed to matches), the backgammon player should determine his maximum bet, taking into account the following adjustments:

adjustment I: percentage of long-term bankroll which can be risked during a session;

adjustment II: number of games to be played;

adjustment III: regulations concerning the doubling cube; and,

adjustment IV: number of possible opponents.

Adjustments to be made for the player who wishes to play matches will be discussed in Section 6.5.

6.1 Adjustment I: Percentage of long-term bankroll which can be risked during a session

Peter explains that, for games in general, a gambler may take different attitudes, approaches, or strategies. Diagram 5 illustrates four types of strategies.

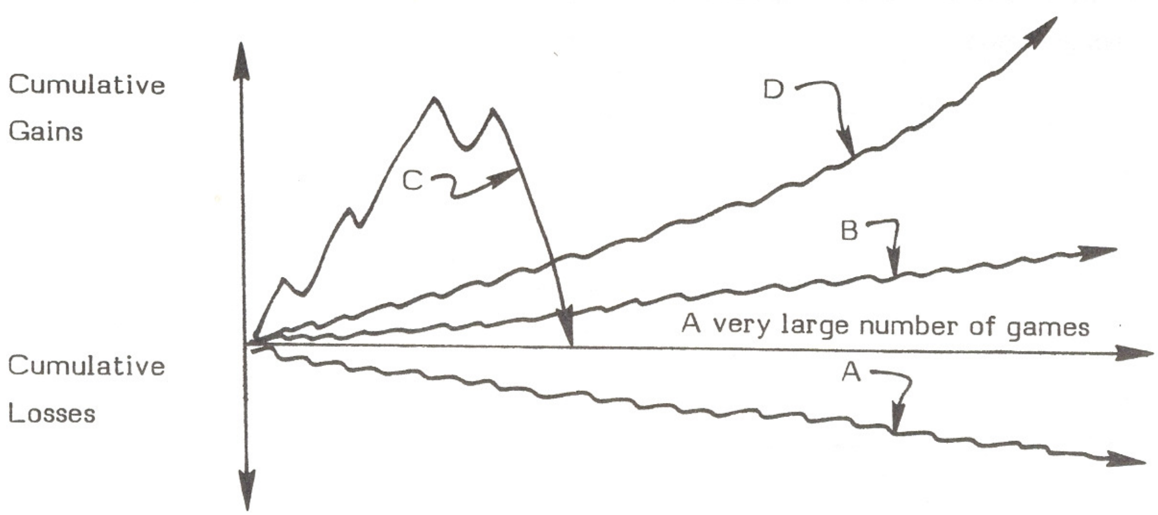


Diagram 5: Illustration of the probable results of different strategies

Strategy A is that of a player who plays in an unfavorable game ($P < 50\%$). If all players are separated into two classes, there will be winners and losers. The percentage of winners should be about 30% to 40%. The percentage of players earning a living with backgammon probably doesn't exceed 2%. In fact, Peter knows over 100 backgammon players, and only one earns his living thereby. The player who plays and loses should bear in mind that he has paid for the pleasure of playing. To be a member of a golf club that charges $2,000 a year in fees or to lose $40 a week at a betting game is strictly a matter of preference. Players who lose regularly, and who still consider that they have had a certain amount of pleasure in proportion to the money they have paid, represent about 60% to 70% of all players. Strategy A reflects the attitude of a player who loses regularly, basically because his single-trial probability of success is less than 50%.

Strategy B is that of a player who plays a favorable game ($P > 50\%$). The gain is proportional to the number of games. This player does not adjust his bet in terms of his cumulative gains. Playing with a uniform bet and not knowing how to increase the bet constitute a weakness.

Strategy C reflects the attitude of a player who forces the game. This type of player takes big risks; if he wins, he wins a lot and, if he loses, he loses heavily. This player, when involved in a favorable game ($P > 50\%$), doesn't apply the fundamental principles of money management. Most of the time, a player like this ends up by meeting a stronger player or by having a "run of bad luck." Because the bet with which this player gambles is too high, he cannot turn his "run of bad luck" around and he loses all his accumulated gains.

Strategy D is used by a player in a favorable game ($P > 50\%$). The player adjusts his bet in terms of his accumulated gains. This player has mastered some money management principles which he follows to the letter. He adjusts his bet to survive "runs of bad luck." Strategy D is a long-term efficient strategy.

Peter explains to John that good money management allows a player to maximize his gains in the long run, and, to attain this objective, it is necessary to have a short-term strategy.

One of the most common errors in money management is to risk more than a reasonable amount in a single-session. For example, a player willing to risk 20% or more of his long-term bankroll in a single-session simply risks too much. The percentage of the long-term bankroll that can be risked during a session should vary from 5% to 15%. Peter believes that 10% seems reasonable. The amount to risk in a session is therefore calculated as follows:

$$\frac{\text{Amount risked}}{\text{(in a session)}} = \frac{\text{Amount to risk}}{\text{long term}} \times \frac{\text{\% to be risked}}{\text{in one session}} \qquad \text{Formula 12}$$

The amount risked as previously established could be wagered if, and only if, the player believes that the opponent has at least the same amount to lose. Practically speaking, one of the main concerns of any player should be to determine how much the opponent is willing to risk. Backgammon is a war and it is perfectly normal to make the effort to know one's opponent. Is the opponent a lawyer, a doctor, a business man or a man on social welfare? How much can he afford to lose (and pay) before he ends the session? A player has to adjust his amount risked (in a session) to the amount that the opponent is willing to risk. The maximum amount of money that a player should be willing to risk should never be over the amount that the opponent could afford to lose (and pay). For example, a player having a potential single-session bankroll of $300 should be willing to risk only $100 if he believes that the opponent has only $100 to risk.

Peter stresses that when John is willing to risk 10% of his long-term bankroll in one session and believes that the opponent will pay whatever amount is lost, he has to quit if he loses this 10%. John asks why? Peter replies "Well, when a player loses what he can afford to lose in one session (i.e. his 10%), he often acts emotionally (i.e. steaming). Consequently, it is good to quit and come back a few days later".

6.2 Adjustment II: Number of games to be played

Peter stresses that another factor to be considered when determining the maximum bet for a backgammon player is the number of games that the player wants to play. When a player gambles at the casino, he is perfectly free to quit whenever he wants to, but, in backgammon, a player is often obliged to play a minimum number of games. Even if, in some cases, the loser can quit any time, Peter believes that the player should determine his maximum bet so as to be able to play a specific number of games.

In backgammon, it is mathematically impossible to establish precisely the single-trial probability of success as this probability changes with time. In fact, if the opponent steams when he loses, it follows that one's single-trial probability of success will now increase. If a player foresees that his opponent will start to steam and modify his style of play after losing 10 points, or that his opponent will lose his concentration after playing 30 games, then he could determine the size of his bet so as to be reasonably certain to play 40 or 50 games. Peter offers Diagram 6 to illustrate the possible consequences of losing emotional control as time passes.

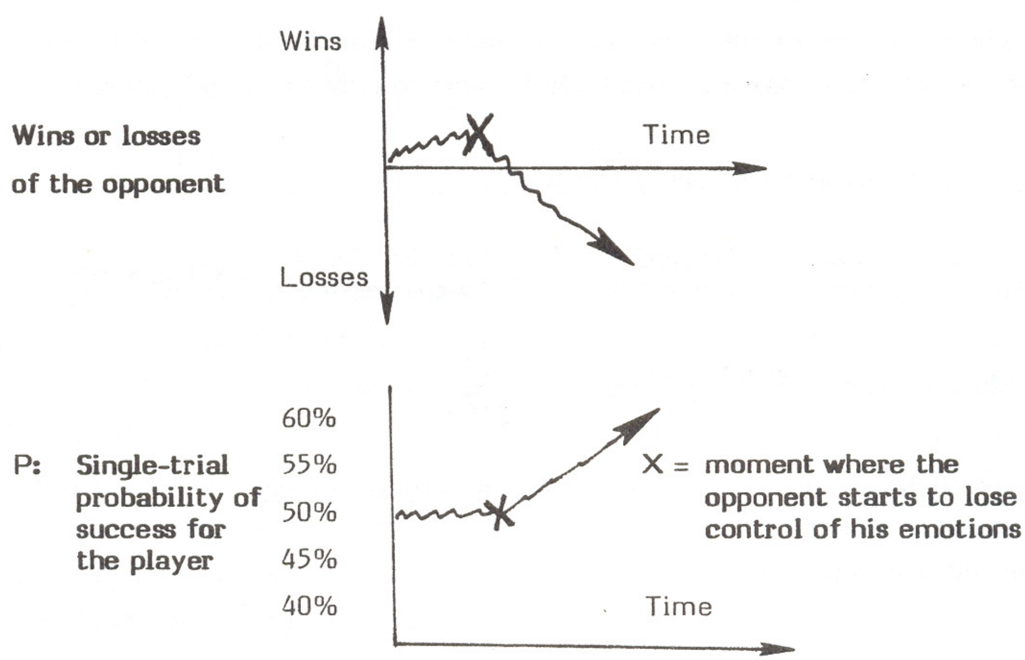


Diagram 6: Possible consequences due to a loss of emotional control by an opponent

The number of games played in a average session can vary greatly from one player to the next. For a gambler who plays six to eight games an hour, 50 games would represent quite a session. On the other hand, for players who are used to a rate of 15 games an hour, a 100-game session might seem perfectly normal. The strict minimum of games to be played for a specific session should be at least 20 games so as to diminish the effect of luck.

Based on Formula 11, Peter suggests the following formula to take into account the number of games to be played:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}}} \qquad \text{Formula 13}$$

N_{prob} = probable number of games to be played (minimum = 20)

Example 25:

Dick and Phil are at approximately the same level of skill. Dick has a $1,000 long-term bankroll, and the percentage he is willing to risk in one session is 10%. Dick knows Phil well and is certain that there will be no problem concerning payment. Both players agree to play without the cube, gammons or backgammons (triple games). What is the maximum bet if Dick wants to play about 100 games?

Using Formulas 12 and 13, John obtains:

$$\begin{matrix}\text{Amount risked} \\ \text{(in one session)}\end{matrix} = \begin{matrix}\text{Amount to risk} \\ \text{long term}\end{matrix} \times \begin{matrix}\text{\% to be risked} \\ \text{in one session}\end{matrix} = \$1000 \times 10\% = \$100$$

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}}} = \frac{\$100}{\sqrt{100}} = \$10$$

To obtain his goal, the maximum bet should be $10 per game.

6.3 Adjustment III: Regulations concerning the doubling cube

Peter now reviews the fact that the regulations of the cube affect the basic bet. Because of the doubling cube and the possibility of gammons and backgammons (triple games), the true average bet is always above the basic bet. For example, the use of "automatic" doubles increases the basic bet. The factor of the increased basic bet depends, among other factors, on the number of automatic doubles allowed (agreed upon by the players). Peter gives these theoretical factors in Table 7.

Table 7

Theoretical Increase Factors for the Automatic Cube

Number of Automatic Cubes	Theoretical Calculations	Theoretical Increase Factors
1	(5/6) x 1 + (1/6) x 2	1.167
2	(5/6) x 1 + (5/36) x 2 + (1/36) x 4	1.222
3	(5/6) x 1 + (5/36) x 2 + (5/216) x 4 + (1/216) x 8	1.241
4	(5/6) x 1 + (5/36) x 2 + (5/216) x 4 (5/1296) x 8 + (1/1296) x 16	1.247
5	(5/6) x 1 + (5/36) x 2 + (5/216) x 4 + (5/1296) x 8 + (5/7776) x 16 + (1/7776) x 32	1.249

Peter explains that, **theoretically speaking,** if John plays a very large number of games (for example, 1000 games) he will play with a true initial bet of 1.167 times the basic bet if he plays with one automatic double, with 1.222 times the basic bet if he plays with two automatic doubles and so on. But John replies that, **in practice,** the initial bet can be twice the basic bet if one automatic double is allowed, or four times the basic bet if two automatics are allowed, and so on. Peter stresses that if we call "Nd" the number of automatics doubles allowed, then the possible level of the cube at the beginning of the game is:

$$\text{Possible level of the cube at the beginning of the game} = 2^{Nd} \qquad \text{Formula 14}$$

Nd = number of automatic doubles allowed

Peter points out that 2 to the power 0 is equal to 1 ($2^0 = 1$). In other words, if there are no automatic doubles allowed, then the level of the cube at the beginning of each game (according to Formula 14) is 1.

Peter explains that, since the player who permits five automatic doubles is exposed to the possibililty of beginning a game with the cube at 32, and because the theoretical increase factor is only 1.249, it seems very logical to use a factor somewhere between those two extremes to evaluate the effect that automatic doubles can have on the bet. The suggested factor that will take into account the number of automatic doubles is:

$$\text{Automatic cube factor} = 1.17^{Nd} \qquad \text{Formula 15}$$

Nd = number of automatic doubles allowed

* * * * *

Peter stresses that the cube is an essential factor to be considered and that each player has his own way of handling the cube. It is therefore necessary for each player to establish his own "Cube Factor", defined as follows:

$$\text{Cube Factor} = \frac{\text{Number of points}}{\text{Number of games}} = \frac{\text{True average bet}}{\text{Basic bet}} \qquad \text{Formula 16}$$

For example, Peter's cube factor while playing with no automatic doubles varies between 1.5 and 2.5. As a guide, Peter prepares Table 8 which gives some "reasonable" cube factors.

Table 8

Example of "reasonable" cube factors

2.0:	with no automatic doubles
2.3:	with one automatic double
2.7:	with two automatic doubles
3.6:	with no automatics and rule No 1 (below)
4.2:	with one automatic and rule No 1
4.9:	with two automatics and rule No 1
6.0:	with no automatics and rule No 2 (below)
7.0:	with one automatic and rule No 2
8.2:	with two automatics and rule No 2

Rule No 1: The player winning the opening roll can refuse it and turn the cube to the next level. The player must play his second roll if it is not a double; if it's a double, he rolls again, with the cube staying at the same level. This rule is also called the "California Doubles".

Rule No 2: The player winning the opening roll can refuse it and turn the cube to the next level. The player must play his second roll if it is not a double; if it's a double, he rolls again, with the cube staying at the same level. If the player has refused his first roll, then the opponent has the same privilege and can refuse his first roll and turn the cube to the next highest level. The opponent could play the double he obtains.

Table 8 is constructed on the assumption that, with no automatic doubles, the cube factor equals 2.0. With one automatic double, the cube factor is evaluated at $2.0 \times 1.17^1 = 2.3$, and with two automatic doubles the cube factor is evaluated at $2.0 \times 1.17^2 = 2.7$.

Concerning Rule No 1, Peter points out that the only rolls that should be accepted are 3-1, 4-2 and 6-1; with all other rolls, it is in the player's interest to turn the cube to the next highest level. Because doubles are excluded for the opening move, there are 30 possibilities. With six of those possibilities, the cube remains at the same level and, in the other 24 possibilities, the cube increases a level. Therefore, Rule No 1 has an increasing effect of (6/30) (1) + (24/30) (2) = 1.8. With no automatic doubles, the cube factor is evaluated at 2.0, therefore, with no automatic doubles <u>and</u> the use of Rule No 1, the cube factor is evaluated at 2.0 x 1.8 = 3.6.

Regarding Rule No 2, Peter points out that, out of the 36 possibilities, the good ones are 3-1, 4-2, 6-1, 1-1, 2-2, 3-3, 4-4, 6-6 (5-5 may be good or not); so let's say there are 12 good possibilities and 24 bad ones. Thus, Rule No 2 has an increasing effect of (12/36) (1) + (24/36) (2) = 1.67. With no automatic doubles <u>and</u> Rule No 2, the cube factor is evaluated at 2.0 x 1.8 x 1.67 = 6.0.

Peter notes that the use of Rules 1 and 2 during a session will probably predispose the players to move the cube up to the higher levels when the player who wins the opening roll accepts it. One could expect, therefore, more early cubes, beavers, etc. In this way the "true" cube factor may be a little higher than that previously established. Peter and John agree on the fact that the best way to establish the cube factor remains an evaluation based on a compilation of games already played.

To determine the maximum bet in relation to the number of games to be played and the regulations concerning the doubling cube, Peter suggests the following formula:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}} \times C_f} \qquad \text{Formula 17}$$

N_{prob} = probable number of games to be played

$$C_f = \text{Cube factor} = \frac{\text{Number of points}}{\text{Number of games}} = \frac{\text{True average bet}}{\text{basic bet}}$$

Example 26:

Dick agrees to play a series of 10 games with another player. When a series is finished, the payment is made, and a new series may start if both players agree. After playing two series, Dick obtains the following scoresheet:

1st series: +1, 0, +2, 0, +1, +2, +4, +3, +5, +4

2nd series: -1, +1, -1, +3, +1, +5, +6, +4, 0, -2

What is the cube factor?

The simplest way to calculate the cube factor is to calculate the number of games ended by 1 (or -1), by 2 (or -2) and by 4 (or -4) and proceed as follows:

Number of games ended at level 1 (8 games x 1 point)	:	8 points
Number of games ended at level 2 (8 games x 2 points)	:	16 points
Number of games ended at level 4 (4 games x 4 points)	:	16 points
Total	:	40 points

The total number of points (won or lost) is 40, and 20 games were played. The cube factor is 40 points/20 games = 2.0. This means that the true average bet represents 2.0 times the basic bet. If the basic bet is $10 a point, Dick is, in fact, playing at $20 a game.

Example 27:

Dick and Phil are at the same skill level. Dick has a $2,000 long-term bankroll; the percentage he is willing to risk in this session is 10%. Dick knows that he won't have any problem to be paid if he wins. For this session, Dick would like to play a strict minimum of 30 games. He has already established his cube factor at level 2.0 playing with no automatic doubles. Phil insists on playing with 2 automatic doubles and Dick accepts. What is the maximum bet?

The amount risked for this session is $2,000 x 10% = $200 (Amount risked = $200). Since Dick wants to play a minimum of 30 games, it is necessary to increase, by a certain amount, the number of games to be played. We can fix the number of games to be played at 50 ($N_{prob} = 50$). With a cube factor established at 2.0 when no automatic doubles are permitted, the correct cube factor, taking into account two automatic doubles, is then $2.0 \times 1.17^2 = 2.7$ ($C_f = 2.7$).

By using Formula 17, we have

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob} \times C_f}} = \frac{\$200}{\sqrt{50 \times 2.7}} = \$10.48$$

The theoretical maximum bet for Dick is $10.48 a point, and the practical maximum bet is $10 a point.

6.4 Adjustment IV: Number of Possible Opponents

Peter emphasizes that the backgammon player should also adjust his bet in relation to the number of opponents he faces. If there is only one opponent, there are no adjustments. If the player competes in a chouette of 3 players, on the average he will be a partner one out of three times, he will be captain one out of three times, and in the box one out of three times. When he is in the box, he will meet two opponents and play 2 times the unit bet. In the case of a chouette with 3 players, the theoretical average bet is established as follows:

$$1/3 \times 1 + 1/3 \times 1 + (1/3) \times 2 = 1.33 \text{ times the unit bet}$$

If a player plays in a chouette of four, the theoretical average bet is:

$$1/4 \times 1 + 1/4 \times 1 + 1/4 \times 1 + (1/4) \times 3 = 1.50 \text{ times the unit bet}$$

If a player plays in a chouette of five:

$$1/5 \times 1 + 1/5 \times 1 + 1/5 \times 1 + 1/5 \times 1 + (1/5) \times 4 = 1.60 \text{ times the unit bet}$$

In a general way, the competitor playing chouette with N players will play from time to time with a number of opponents represented here by "N_o". Peter establishes the theoretical relation between the true average bet and the unit bet as follows:

$$\text{True average bet} = \frac{2\,N_o}{N_o + 1} \times \text{unit bet} \qquad \text{Formula 18}$$

N_o = Number of opponents

By using this formula, the following results are obtained:

Table 9

Theoretical Increase Factor in Relation to the Possible Number of Opponents (having the same level of skill)

Possible Number of Opponents	Theoretical Increase Factors
1	1.00
2	1.33
3	1.50
4	1.60
5	1.67

Peter stresses that, **theoretically speaking,** a player who plays a large number of games (for example 1,000 games) with players of equal strength will play with a true bet of 1.33 times the basic bet if he plays against two opponents (chouette of 3), at 1.50 if he plays against three opponents (chouette of 4), and so on. But, in **practice,** Peter believes that the player who plays in a chouette of 3 should divide by 2 his pre-established maximum bet obtained by playing only one opponent; in the same way, the player who plays in a chouette of 4 (and thus plays against 3 opponents) should divide by 3 his maximum bet. Peter concedes that this might be considered by some to be a conservative approach, but it's the one he prefers.

* * * * *

To determine the maximum bet in terms of the number of games to be played, the regulations of the cube and the number of possible opponents, Peter suggests the following formula:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob} \times C_f \times N_o}} \qquad \text{Formula 19}$$

N_{prob} = probable number of games to be played

C_f = cube factor

N_o = number of opponents

Example 28:

Dick plays in a chouette of 4 ($N_o = 3$). The opponents are of equal strength. Dick has $3,000 long-term bankroll and the percentage at risk in this session is 10%. Dick doesn't expect any problem in regard to payment. For this session, he wants to play about 60 games. The cube factor is evaluated at 2.0. What is the maximum bet?

In using Formula 19, we obtain:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}} \times C_f \times N_o} = \frac{\$3000 \times 10\%}{\sqrt{60} \times 2.0 \times 3} = \$6.45$$

The maximum theoretical bet is $6.45 a point, and the practical maximum bet for Dick is $5 a point.

* * * * *

Once the maximum bet has been established with Formula 19, the player has the choice of which criterion to use to determine his appropriate bet. Peter recalls that, if the player wants to maximize his probability of success, he has to play with the minimum bet if the game is favorable ($P > 50\%$) and with the maximum bet if the game is unfavorable ($P < 50\%$). If the player wants to maximize his hourly expectation, he must play with the maximum bet if the game is favorable ($P > 50\%$) and with the minimum bet if the game is unfavorable ($P < 50\%$). The minimum bet is the smallest possible bet that all parties agree to.

Example 29:

Dick has a long-term bankroll of $3,000 with which to play backgammon. For the session he is currently playing, however, he can only risk the $300 in his pockets. Dick plays against Phil and knows that there won't be any problem with payment. Phil is of equal skill but tends to steam when he loses, thus becoming a weaker player. For this reason, Dick believes his single-trial probability of success is superior to 50% ($P > 50\%$). He foresees that he will probably play 50 games ($N_{prob} = 50$). The players use the doubling cube with no automatic doubles, the cube factor being evaluated at 2.0 ($C_f = 2.0$). What is the theoretical maximum bet? What is the practical maximum bet? What is the appropriate bet if Dick wants to maximize his probability of success? What is the appropriate bet if Dick wants to maximize his hourly expectation?

By using Formula 19, we have:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}} \times C_f \times N_o} = \frac{\$300}{\sqrt{50} \times 2.0 \times 1} = \$21.21$$

The theoretical maximum bet for Dick is $21.21; the practical maximum bet is $20. If Dick wants to maximize his probability of success, he must play with the minimum bet, which could be $1 or $2, depending on the opponent. If, on the other hand, Dick wants to maximize his hourly expectation, he should play with the maximum bet, i.e. $20 a point.

* * * * *

Peter explains to John that, when the game is favorable, it is possible for a player to establish his bet so as to have a pre-established probability of success. In this case, he should use Formula 10 and modify it for backgammon. Peter suggests the following formula:

$$\text{Appropriate bet} = \frac{\text{Amount risked} \times \text{Log}(Q/P)}{C_f \times N_o \times \text{Log}(1-P_{success})} \qquad \text{Formula 20}$$

C_f	=	Cube factor
N_o	=	Possible number of opponents
P	=	Single-trial probability of success
Q	=	Single-trial probability of ruin (Q = 1-P)
$P_{success}$	=	Probability of winning the amount desired to win instead of losing the amount to risk

Example 30:

After having played a large number of games with the same opponent, Dick establishes his single-trial probability of success at 54% (P = 54%). The cube factor is estimated at 2.0 (C_f = 2.0). Dick has a $2000 long term bankroll and, for this session, he has 10% of this to risk (amount risked = $200). Knowing that his opponent can afford to lose more than $200, he wants to win $200 or more. With what bet should he play to obtain a probability of success of about 80%?

By using Formula 20 with $P_{success}$ = 80%, we have:

$$\text{Appropriate bet} = \frac{\$200 \times \text{Log}(.46/.54)}{2.0 \times 1 \times \text{Log}(1-.8)} = \$9.96 \approx \$10$$

The appropriate bet, then, is $10.

Example 31:

After playing 50 games at $10 a point, Dick wins 10 points and is paid. His opponent has lost control of himself (i.e. he is steaming, or "on tilt") and suggests that they play one game at double or nothing for $100, with no cube, gammons or backgammons. The opponent shows Dick that he has the money. Dick estimates his chances of winning (each game) at 60% (P = 60%). Since he does not find this proposition interesting enough, he would like to suggest a bet that will give him a probability of success (winning $100 instead of losing $100) of about 80%. What is the appropriate bet which will permit him to attain this goal?

Formula 20 is used:

$$\text{Appropriate bet} = \frac{\text{Amount risked} \times \text{Log}(Q/P)}{C_f \times N_o \times \text{Log}(1-P_{success})}$$

Since there is no cube, no gammons, no backgammons, the cube factor is equal to 1.

The appropriate bet is calculated as follows:

$$\text{Appropriate bet} = \frac{\$100 \times \text{Log}(0.4/0.6)}{1.0 \times 1 \times \text{Log}(1-.8)} = \$25.19$$

If Dick suggests playing at $25 a game (no cube, gammon, backgammon) until one of them loses $100, then he has obtained a probability of success around 80%.

* * * * *

Peter notes that the fundamental principle of money management doesn't teach a player to play with a constant and uniform bet. On the contrary, the fundamental principles of money management state quite clearly that the bet is a function of the single-trial probability of success. Therefore, if that probability changes during the course of time, the bet can also change. The next example illustrates this point.

Example 32:

Dick plays an opponent against whom the single-trial probability of success (established after a large number of games) is 54%. He estimates at the beginning of the session that the single-trial probability of success is 52%, at the middle of the session it is 54% and, in the last part of the session, it is 58%. It rises this high, if his opponent is losing on the score sheet, because he steams and accepts bad cubes. Dick has $200 to risk for the session and wants to win a similar amount. He knows that the opponent can afford to lose more than $200. The cube factor is evaluated at 2.0. Knowing that his opponent will accept the bet proposed to him, and will also accept increased bets, even if he is losing, what is the best strategy for Dick to use?

If Dick wants to have a probability of success of about 80%, then the results obtained using Formula 20 are:

P	**Theoretical Bet**	**Practical Bet**
52%	$ 4.97	$ 5.00
54%	$ 9.96	$10.00
58%	$20.06	$20.00

At the beginning of the session, when P is approximately 52%, the bet can be $5 a point. If, in the middle of the session, P approximates 54%, then the bet can be increased to $10 a point at the end of the session if the opponent is losing, and, if P is estimated at approximately 58%, then the bet can be increased to $20 a point.

* * * * *

Theoretically, it is possible to justify an increase of the bet during the same session, but to do so the player should also take his sessional expectation into consideration. In fact, it might also be correct, in practice, to refuse to increase the bet if a player believes that his opponent will steam even more and lose more as a result.

6.5 Playing matches

If a player wishes to play matches, rather than games, then he may use Formula 19 to evaluate the maximum amount he could bet. To do this, it must be supposed that the cube factor and the number of opponents are both equal to 1 ($C_f = 1$ and $N_o = 1$). In this case, N_{prob} indicates the probable number of matches that the player wishes to play. For example, if he wishes to play four matches per session, he should divide his single-session bankroll by two; if he wishes to play nine matches in a single-session, then he should divide his single-session bankroll by three and, finally, if the player wishes to play 16 matches in a session, then he must divide by four.

It's also possible for the player to set his bet in such a way as to obtain a pre-established probability of success. In this case, he must use Formula 20 with a cube factor and number of opponents both equal to 1 ($C_f = 1$ and $N_o = 1$). The single-trial probability of success represents the probability of winning every match.

The player should bear in mind that he must play a minimum of about 20 games in order to diminish the luck factor. In a session, therefore, the strict minimum is:

- one match of 13 points, or
- two matches of 7 points

A player may be justified in not playing if the opponent does not agree to this minimum.

Example 33:

Dick meets an opponent who insists on playing matches rather than playing games. Dick has a long-term bankroll which he estimates at $2,000 and a single-session bankroll evaluated at $200. The opponent can afford to lose the latter amount.

A) Dick wishes to play about 4 matches of 7 points. Dick believes that he is better than his opponent and he wishes to maximize his hourly expectation. What is the appropriate bet?

Dick must evaluate his maximum bet using Formula 19 (with $C_f = 1$ and $N_o = 1$).

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob} \times C_f \times N_o}} = \frac{\$200}{\sqrt{4 \times 1 \times 1}} = \$\,100.00$$

The maximum theoretical bet is $100.00 per match and the maximum practical bet is $100 per match. Since Dick believes he is better than his opponent, and since he wants to play to maximize his hourly expectation, he should play at $100 a match.

B) Dick wishes to win an amount greater than $200 with a probability of success of about 80%. Dick estimates that he has a probability of 60% to win every match. What is the appropriate bet?

Dick must use Formula 20 (with $C_f = 1$ and $N_o = 1$).

$$\text{Appropriate bet} = \frac{\text{Amount risked} \times \text{Log}(Q/P)}{C_f \times N_o \times \text{Log}(1-P_{success})}$$

$$\text{Appropriate bet} = \frac{\$200 \times \text{Log}(.4/.6)}{1 \times 1 \times \text{Log}(1-0.8)} = \$50.39$$

Therefore, to obtain a probability of success of about 80%, the appropriate bet is $50 a match.

CHAPTER 7 - COMPLETE STRATEGY SUGGESTED FOR BACKGAMMON

To determine the appropriate bet for a backgammon player, Peter points out that there are three steps to be taken:

step 1: evaluation of the maximum bet;

step 2: evaluation of the single-trial probability of success; and,

step 3: choice of criterion to use.

7.1 Evaluation of the maximum bet

To establish the maximum bet, the player has to determine the following variables:

- long term bank roll
- amount to risk in one session (Amount risked)
- probable number of games to be played (N_{prob})
- cube factor (C_f)
- number of opponents (N_o)

Once all these variables are considered, the player can calculate his own maximum bet using Formula 19:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}} \times C_f \times N_o}$$

The amount risked should not be greater than the amount that the opponent can afford to lose.

7.2 Evaluation of the single-trial probability of success

Peter points out that the objective evaluation of the single-trial probability of success (P), which is to say, the evaluation of the relative strength of the opponent to the player's strength, is often a difficult one. The best way to evaluate P is to play with the opponent, keeping a record of the results and to analyze the information afterwards. Example 12 indicates how to do this.

There are also other ways to evaluate P. John can, for example, check an opponent while he is playing with others. If John is not able to recognize any major mistakes, then he can assume that this player is at least as good as he is. He then evaluates the single-trial probability of success at 50% ($P = 50\%$). If John recognizes some mistakes being made, he can assume that his single-trial probability of success is above 50% ($P > 50\%$). If he sees this player making imaginative plays which bring positive results, plays that he himself could not have accomplished, or would not have thought of, he can assume that this player is better than he is ($P < 50\%$).

John can also categorize an opponent in terms of the results the opponent obtains against players whose abilities are known. Let's say that player A is known to be superior to player B. Player C (unknown by player A) plays with player B, player B coming out on top. From this fact, player A can deduce that he is superior to player C and, then, that P is above 50% ($P > 50\%$).

John can also accept other people's judgement of a player's skill, but in doing so he should be cautious. Such judgements are often in error. It is also strongly suggested that John not play with his maximum bet against an unknown opponent. It is always much better for a player to establish the value of P himself rather than accept the evaluation of others.

If John meets an unknown player, he can try to get information about him by asking the following questions:

- How long have you been playing?
- Have you read any books on the subject?
- Have you even taken part in a tournament?
- Do you know this or that player?
- Are you accustomed to play with precision dice?

Because people, in general, like to talk about themselves, it is very probable that the opponent will offer information without hesitation.

Peter reiterates that during a given session, as shown in Diagram 6, the single-trial probability of success may change with time. To sum up, the objective evaluation of P is difficult to establish, but it must be done in order to establish the appropriate bet.

7.3 Choice of criterion to use

Once the maximum bet and the single-trial probability of success have been established, the player must then choose which criterion to use. Peter stresses that there are three cases to consider:

A) The single-trial probability of success is superior to 50% (P > 50%):

The appropriate bet for a player who wants to maximize his probability of success is the minimum bet; when the player chooses this criterion, it implies that time is of secondary importance. The appropriate bet for a player who wishes to maximize his hourly expectation is the maximum bet. In the latter case, time is, in a certain sense, the player's enemy; that is to say, the player must confront an opponent and, at the same time, go against the clock. It is also possible to choose a criterion which is a compromise between the two preceding criteria; in effect, the player can play in such a way that he obtains a pre-established probability of success. In this case, the bet is established using Formula 20:

$$\text{Appropriate bet} = \frac{\text{Amount risked} \times \text{Log}(Q/P)}{C_f \times N_o \times \text{Log}(1-P_{success})}$$

B) The single-trial probability of success is inferior to 50% (P < 50%):

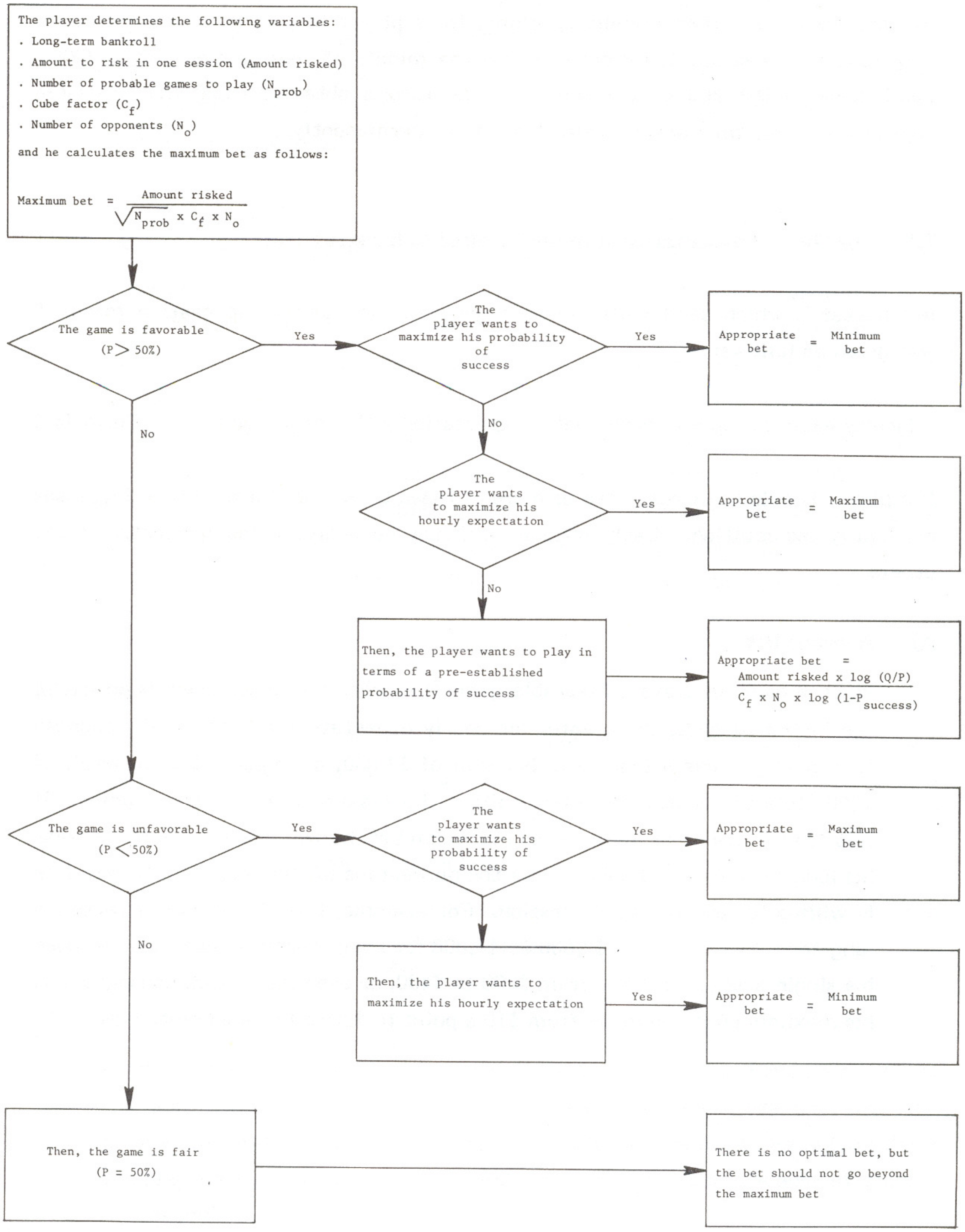
The appropriate bet for the player who wants to maximize his probability of success is the maximum bet, and the appropriate bet for the player who wants to maximize his hourly expectation is the minimum bet. The expression "maximize his hourly expectation" corresponds in reality to "minimize his expectation of hourly losses".

C) The single-trial probability of success is equal to 50% (P = 50%):

When P = 50%, there isn't an optimal bet nor an appropriate bet; there is only one constraint, namely, that the player shouldn't go beyond his maximum bet.

To make a clear summary, Peter presents Diagram 7 entitled "Suggested approach to determine the appropriate bet for a backgammon player".

Diagram 7: Suggested approach to determine the appropriate bet for a backgammon player



Peter points out that the variables of Diagram 7 are subject to change. It follows that the appropriate bet can vary even in the course of a single session. Thus, it can be very justifiable, theoretically speaking, for a player to play at $5 a point at the beginning of a session, $10 a point during the middle of that session and at $20 a point towards the end of the session. The results obtained using the suggested approach in Diagram 7 are, therefore, not fixed permanently.

7.4 The "hourly expectation formula" applied to backgammon

In Chapter 1, which dealt with games in general, the "hourly expectation formula" was given as follows:

Hourly expectation = Average bet x Expectation x Games per hour   Formula 2

If a player is able to increase one or more of these three variables, he then increases his hourly expectation. Each variable will now be analyzed independently of the others.

A) Average bet

When a gambler plays a favorable game (i.e. when the expectation is positive), the higher he bets, the greater his hourly expectation will be. Let's suppose that a player has a long-term bankroll of $2,000, a single-session bankroll of $200 (10% of his long-term bankroll), and a maximum bet of $10 a point. If this player wishes to increase his maximum bet, he will either have to increase his long-term bankroll, or increase the percentage of the long-term bankroll he is willing to risk in a single session. For example, this player can increase his long-term bankroll from $2,000 to $4,000 (or even more); he can also increase his single-session bankroll from $200 to $400, or even more; and, consequently, his maximum bet can move from $10 a point to $20 or more a point.

B) Expectation

The larger the overall expectation, the greater the hourly expectation will be. There are several possibilities for a player who wants to increase his expectation. He can choose to play against weaker opponents. He can also increase his own strength by reading books on the subject and analyzing backgammon problems.

Once an opponent has been chosen, and the session has begun, the player who shows better concentration and better emotional control will succeed in increasing his expectation. For example, John may have an expectation of 2 to 3% at the beginning of a session. However, if he concentrates more than his opponent does, and, if John shows strong self-control, then he may have a 5 to 6% advantage by the middle of the session, and that percentage may increase to 10% towards the end of the session. A player who wishes to maximize his expectation must be patient and demonstrate excellent concentration and self-control. A player may thereby increase his expectation by up to 5 times (from 2-3% to 8-10%) what it was at the beginning of the session. This advice is valid no matter what criterion is used to establish the player's bet.

C) Games per hour

The greater the number of games per hour, the greater will be the hourly expectation. There are two ways for a player to increase the number of games per hour:

1. by playing more quickly (simply by moving the pieces faster);
2. by modifying his approach to cube theory (taking into account that the objective is to maximize the hourly expectation).

1) Moving the pieces faster

Some players complete about 8 games an hour, which is considered slow; those who play about 15 games an hour are considered fast. Players who play quickly are, within reasonable limits, not much more prone to error than those who play slowly. Rather, they are likely to be able to do so because of greater experience.

Often, when a player who is capable of completing 15 games an hour plays against an opponent whose normal rhythm is 8 games an hour, the faster player imposes his rhythm on his opponent. When the rate is established at 10 to 12 games an hour, then the faster player will increase his hourly profit (assuming he is stronger than the other player) because:

a) he plays more games per hour, and
b) his opponent is likely to make more mistakes than usual because he isn't used to playing quickly.

Thus, the strong player who moves his pieces quickly and who imposes his rhythm on an opponent will increase his hourly expectation.

2) Modify your approach to cube theory

A backgammon player can increase his hourly expectation by modifying his approach to cube theory. Until now, books on the subject have conditioned backgammon players to ask themselves the following question when they are in a position to offer the cube:

> "If I play this position 1,000 times, will I make more money by doubling or by not doubling?"

Similarly, to decide if the cube should be accepted or not, the question then becomes the following:

> "If I play this position 1,000 times, will I lose less money by accepting or by refusing the cube?"

A backgammon player can increase his hourly expectation by asking himself the following question:

> "Will I make more money per hour by doubling or by not doubling?"
> "Will I lose less money per hour by accepting or by refusing?"

For a single given position, it is possible to ask oneself several different questions, and it is equally possible to obtain distinctly different results. The next chapter will deal exclusively with this subject.

CHAPTER 8 - MONEY MANAGEMENT VERSUS THE DOUBLING CUBE THEORY

Many gamblers believe that, money management in backgammon is essentially knowing when to offer and when to refuse the cube in relation to: 1) the level of the cube; and 2) the score. Peter explains to John that, beyond the obvious, the most important application of the principles of money management is to determine the appropriate bet in relation to a number of variables and criteria. Once a player has clearly identified the criterion he has been using, he should adapt his "Doubling Cube Theory" to this criterion.

Players who determine their appropriate bet according to the criteria, "maximization of the probability of success" and "the pre-established probability of success", consider that the amount of time used to achieve their goals is secondary. Consequently, these players must attempt to maximize their expectation for each game. Their decision should not, as a rule, depend on either the level of the cube or on the score. Players should handle the cube in a conventional fashion.

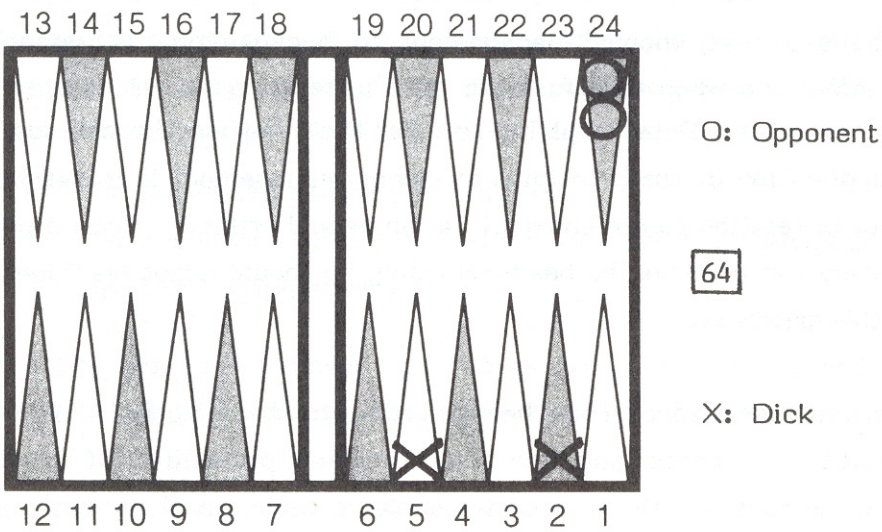
Players who determine their appropriate bet according to the criterion, "maximization of the hourly expectation", consider time to be of importance and should, normally, handle the cube in the following fashion:

1) When the player can double or redouble, he should compare what the hourly expectation would be if the cube were offered as opposed to the cube not being offered.

2) When an opponent offers the cube, the player should compare the hourly expectation he can probably achieve if he refuses the cube with the hourly expectation derived from playing the position as a proposition.

Peter explains to John that a player's criterion, towards the end of a session, may be to maximize the expectation of the session. In other words, the player may have a positive score that satisfies him and he wants to play so as to be sure he'll still have that positive score when the session ends. For that reason, a player might choose not to offer cubes that he would normally give, and not to accept cubes that he would normally take. Some gamblers may argue that this advice is not really relevant to money management. Peter knows that though the definition of money management may vary from one player to the next, the main thing, no matter which definition employed, is the result achieved.

Example 34:



A) Dick wishes to maximize the expectation of this game. Should he offer the cube at level 2? Should he offer the cube, no matter what the level might be?

Dick must ask himself the following question:

> "If I play this position, as a proposition, 1,000 times, will I make more money by doubling or by not doubling?"

This is a classic position and it should be obvious to Dick that he should offer the cube. If he doesn't offer the cube, his expectation is 2/36 of a point (19/36 - 17/36), but his expectation becomes 4/36 of a point if he offers the cube. Dick should offer the cube at any level.

B) Dick wants to maximize his hourly expectation. Should he offer the cube at level 2? Should he offer it at any level?

In this case, Dick must answer this question:

> "If I play this position as a proposition, will I make more money per hour if I offer the cube or if I don't offer the cube?"

By offering the cube, Dick increases his expectation from 2/36 to 4/36 in about 15 seconds, i.e. about 240 plays an hour. If Dick doesn't double, he will win about 13.3 points an hour and, if he doubles, he will win about 26.6 points an hour. Thus, if Dick doubles, he gains about 13.3 points an hour. Dick should offer the cube at any level.

C) Dick is 10 points ahead on the session and would like to maximize his expectation for the complete session. There are now only 10 to 15 minutes left to play. Should he offer the cube at level 2? Should he offer it at any level?

Since Dick now wishes to maximize his sessional expectation, the question becomes the following:

> "If I find myself in this situation 100 times, will I end up with a reasonably positive score more often by doubling or by not doubling?"

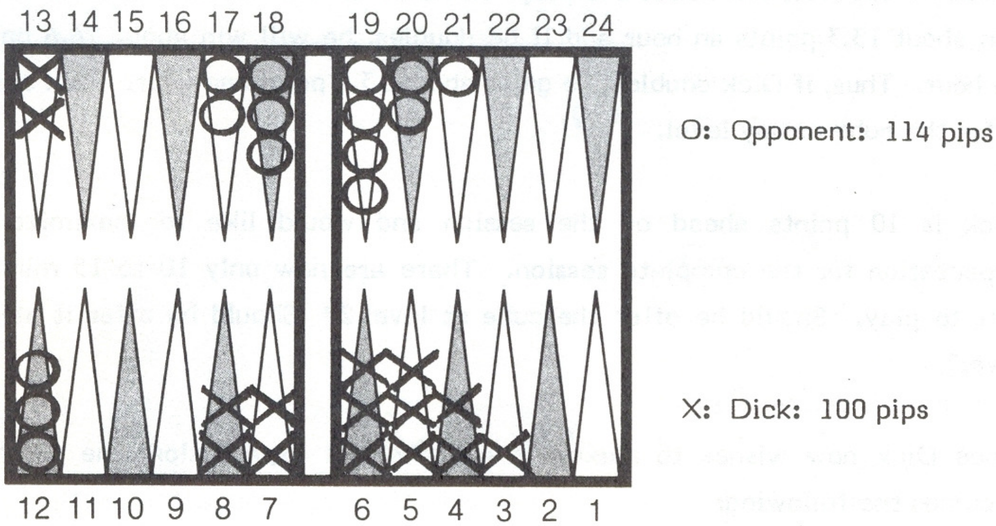
Peter has evaluated the situation in relation to the above criterion, and believes that the cube should be offered at levels 2 and 4, but should not be offered at levels 8 and 16.

For the position discussed above, the results may be recapitulated as follows:

Should a player offer the cube?

Cube level (after being offered)	Player wishes to maximize his overall expectation	Player wishes to maximize his hourly expectation	Player wishes to maximize his sessional expectation
2	yes	yes	yes
4	yes	yes	yes
8	yes	yes	no
16	yes	yes	no

Example 35:



A) Dick wishes to maximize his overall expectation. Should he offer the cube at levels 2, 4, 8, 16?

Dick has an advantage of 14% in the race. This is a classic position and, in order to maximize the overall expectation, all books state that the cube must be given. The cube should then be offered at any level.

B) Dick wishes to maximize his hourly expectation. If he offers the cube, his opponent will take at least one minute to assess the race and will probably take it. From that moment, it will take about three minutes to play to the end. Dick knows that, if his advantage in the race increases from 14% to 16%, his opponent will drop out. Should Dick offer the cube at levels 2, 4, 8, 16?

Dick should compare his hourly expectation if he offers the cube with the expectation if the cube is not offered.

If Dick offers the cube at level 2, his opponent will take one minute to assess the race, he will accept the cube and it will then take three minutes to play the position until the end. The expectation for Dick will be about 0.9 of a point and the time to play out this position will be four minutes i.e. 15 plays an hour. Therefore, if Dick offers the cube at level 2, he will win about 13.5 points an hour.

If Dick leaves the cube at level 1, he expects that one minute later he will win about 60% of the games by doubling his opponent. The remaining 40% will be evenly split and will take about three minutes each to complete. If this position is played 100 times, Dick estimates that the time required is 60 x 1 minutes + 40 x 3 minutes = 180 minutes. Therefore, in one hour, that position can be played 33 times. A computer analysis (with no cube) indicates that for the position X: 100 pips, O: 114 pips; X has a probability of winning around 80%. Therefore, the expectation, if Dick doesn't offer the cube, is about 80% x (1 point) + 20% (-1 point) = 0.6 of a point per game. The expectation is about 0.6 of a point per game and there are 33 games an hour. Therefore, if Dick doesn't offer the cube, he will win about 20 points an hour.

If Dick offers the cube, he will have an hourly expectation of about 13.5 points, and, if he doesn't offer the cube, his hourly expectation is about 20 points. Therefore, Dick should not offer the cube and should wait a few moves. This approach is valid for any level of the cube.

C) Dick is 10 points ahead on the session and would like to maximize his expectation for the complete session. There are now only 10 to 15 minutes left to play. Should Dick offer the cube at levels 2, 4, 8, 16?

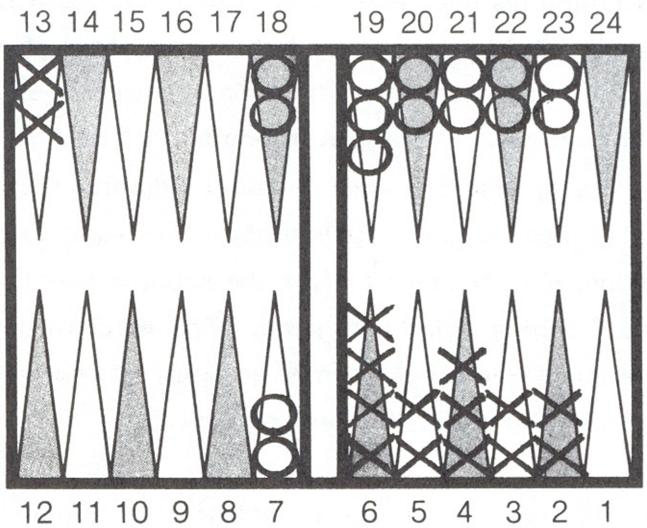
Dick wants to maximize his sessional expectation. Peter believes that the cube should be offered at level 2, but should not be offered at levels 4, 8 and 16 because the probability of being obliged to accept it back at a higher level is too great.

For the position discussed above, the results may be recapitulated as follows:

Should a player offer the cube?

Cube level (after being offered)	Player wishes to maximize his overall expectation	Player wishes to maximize his hourly expectation	Player wishes to maximize his sessional expectation
2	yes	no	yes
4	yes	no	no
8	yes	no	no
16	yes	no	no

Example 36:



O: Opponent: 96 pips

X: Dick: 82 pips

A) Dick wishes to maximize his overall expectation. Should he give the cube at levels 2, 4, 8, 16?

If Dick doesn't give the cube while it is at level 1, his expectation is evaluated as follows:

1) He could lose the game by leaving a shot and being hit immediately. If he is not hit immediately, he will give the cube and the opponent will drop. The probability of leaving a shot is 10/36 (all 6 except 6-6) and the probability of being hit is 17/36. Therefore, the probability of losing the game by being hit is about 13% (10/36 x 17/36).
2) Dick could lose the race but because he will have the opportunity of giving the cube later on, let's make a rough estimate that 10% of the games will be lost in the race.
3) Dick has a probability of winning of about 77% (i.e. 100% - 13% - 10%).
4) The expectation is about (.77 - .23) x 1 point = .54 of a point.

If Dick gives the cube at level 2, he knows that the opponent will correctly accept. He estimates that his expectation may be evaluated as follows:

1) He could lose the game by leaving a shot and being hit immediately (13% of the time) or by being obliged to leave the same blot a second time (with 1-1, 1-2, 1-3, 1-4, 1-5, 2-2, 2-3, 2-4, 3-3, i.e. 15/36) and being hit (1/3 of the time). This last event will occur about 4% of the time (10/36 x 15/36 x 1/3). Therefore, the probability of being hit is about 17%.
2) He could lose the race but, because the game will be played until the end, let's make a rough estimate that the game will be lost 15% of the time.

3) Dick has a probability of winning of about 68% (i.e. 100% - 17% - 15%).

4) Dick's expectation is about (.68 - .32) x 2 points = .72 of a point.

Therefore, if Dick doesn't give the cube, he "loses his market" because he will win about .54 of a point instead of .72 of a point. Therefore, he should give the cube at any level.

B) Dick wishes to maximize his hourly expectation. Should he give the cube at levels 2, 4, 8, 16?

If Dick does not give the cube, the game will end in one minute and a half or less. There will be, then, about 40 games an hour. In this case, his expectation will be about .54 of a point and his hourly expectation will be .54 x 40 games = 21.6 points. If Dick gives the cube at level 2, the opponent will take it and Dick will have an expectation of about 0.72 of a point. The time needed to end the game will be about 3 minutes. This position can be played 20 times an hour, and then the hourly expectation is about 14.4 points. Therefore, Dick should not offer the cube and should wait a few moves. This approach is valid for any level of the cube.

C) Dick is 10 points ahead in the session and would like to maximize his sessional expectation. There are only 10 to 15 minutes left to play. Should he offer the cube at levels 2, 4, 8, 16?

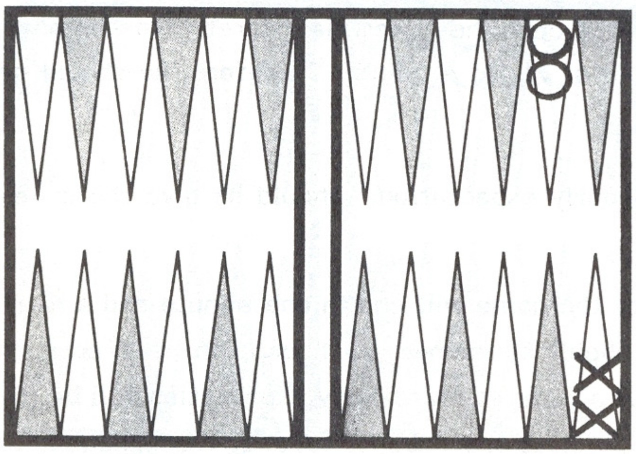
Peter believes that the cube should be offered at levels 2 and 4, but should not be offered at levels 8 and 16.

For the position discussed above, the results may be recapitulated as follows:

Should a player offer the cube?

Cube level (after being offered)	Player wishes to maximize his overall expectation	Player wishes to maximize his hourly expectation	Player wishes to maximize his sessional expectation
2	yes	no	yes
4	yes	no	yes
8	yes	no	no
16	yes	no	no

Example 37:



The opponent offers the cube at level 2, 4, 8, 16.

A) Dick wishes to maximize his overall expectation. Should he accept the cube?

Dick must ask himself:

> "If I play this position 1,000 times, will I make more money (or lose less money) by accepting or refusing the cube?".

This is a classic position. It is obvious that the opponent has a probability of 26/36 of winning this game. If Dick refuses the cube when it's offered at level 2, he loses one point. If Dick accepts it, his expectation is 26/36 x (-2 points) + 10/36 x (+2 points) = -.89 of a point. By accepting the cube, therefore, he gains 0.11 point per game. Dick should accept the cube at any level.

B) Dick would like to maximize his hourly expectation. Knowing that past (and expected) performances are worth five points an hour, should he accept the cube?

Since Dick wishes to maximize his hourly expectation, he must ask himself the following question:

> "If I play this position as a proposition, will I make more money per hour if I accept the cube or if I refuse it?"

If Dick refuses the cube and plays with his opponent, he then has an hourly expectation of five points an hour. If Dick accepts the cube at level 2, he will win 0.11 of a point in about 15 seconds. Since that position could be played at least 240 times an hour, the hourly expectation by accepting the cube at level 2 becomes about 26 points an hour. Dick should, then, accept the cube at any level because, by accepting it, he will make more money an hour than by refusing it.

C) Dick is 10 points ahead on this session, with 10 to 15 minutes left to play. Dick wishes to maximize his expectation for the session. Should he accept the cube?

Since Dick wishes to maximize his sessional expectation, the question becomes the following:

> "If I find myself in this situation 100 times, will I end up with a reasonably positive score more often by accepting or by refusing?

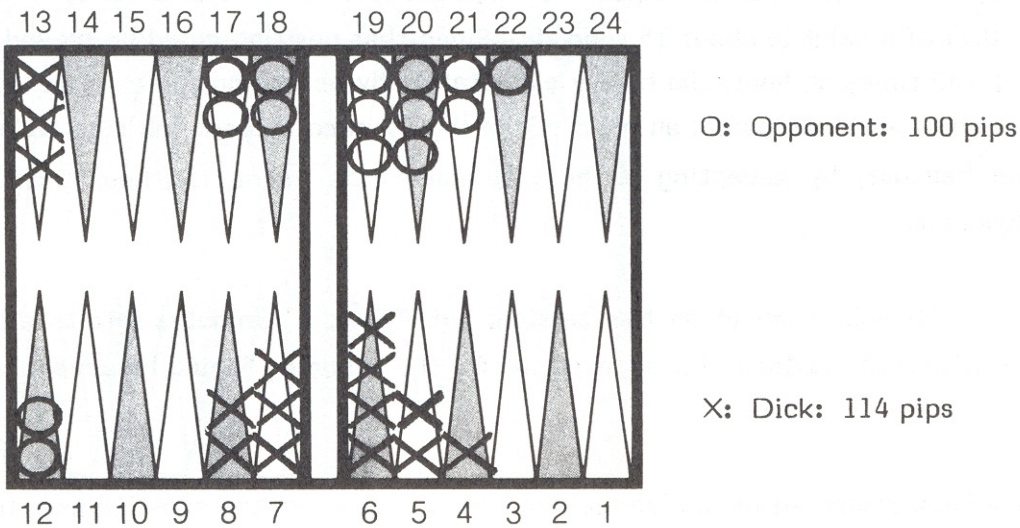
Peter believes that the cube should be accepted at levels 2 and 4, but should be refused at levels 8 and 16.

For the position discussed above, the results may be recapitulated as follows:

Should a player accept the cube?

Level of the cube offered	Player wishes to maximize his overall expectation	Player wishes to maximize his hourly expectation	Player wishes to maximize his sessional expectation
2	yes	yes	yes
4	yes	yes	yes
8	yes	yes	no
16	yes	yes	no

Example 38:



The opponent offers the cube at the following levels: 2, 4, 8, 16.

A) Dick wishes to maximize his overall expectation. Should he accept the cube?

Dick must play according to the "conventional" cube theory. Dick figures that, when the cube is offered at level 2, he will lose about 0.9 of a point, if he accepts, as opposed to losing 1.0 point, if he refuses. Dick must, therefore, accept the cube whether it is offered at levels 2, 4, 8 or 16.

B) Dick would like to maximize his hourly expectation. Knowing that past (and expected) performances are worth five points an hour, should he accept the cube?

Dick must compare his hourly expectation, if he accepts, with his hourly expectation, if he refuses.

If Dick refuses the cube and plays with his opponent, he then has an hourly expectation of five points an hour. If Dick accepts the cube at level 2, he will win 0.1 point per game (lose 0.9 of a point instead of losing 1.0 point). Supposing that this position takes three minutes to play out, it would mean about 20 games an hour. Dick would then make about 2.0 points an hour, playing this position as a proposition. He should, therefore, pass and move on to the next game. If the cube is offered at level 4, Dick should also pass because, playing this position as a proposition, he would win about 4.0 points an hour. However, if the cube is offered at level 8 or higher, Dick should accept because, playing this position as a proposition, he will win more than five points an hour. Therefore, Dick should refuse the cube if it is offered at levels 2 and 4, but he should accept it if it is offered at level 8 or higher.

C) Dick is 10 points ahead on this session, with 10 to 15 minutes left to play. Dick wishes to maximize his expectation for the session. Should he accept the cube?

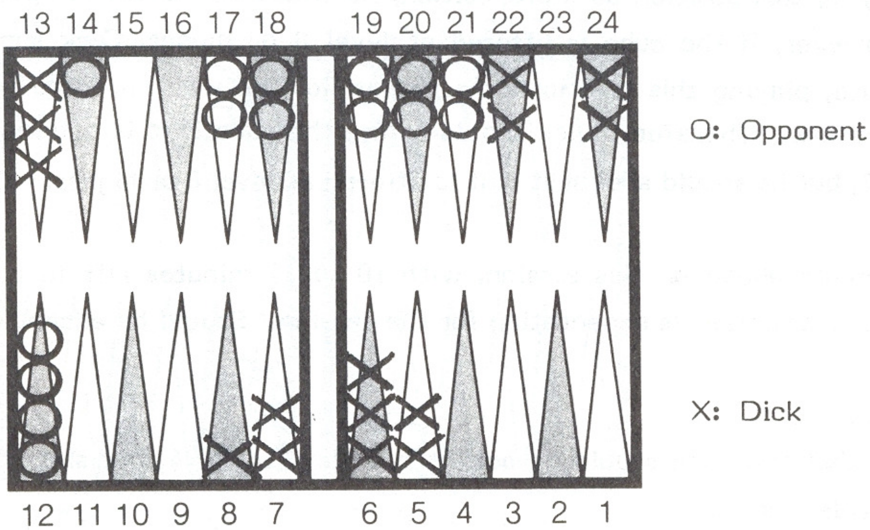
Peter thinks that the cube should be accepted at levels 2 or 4, but should be refused at levels 8 or 16.

For the situation just analyzed, the possible results can be summarized as follows:

Should a player accept the cube?

Level of the cube offered	Player wishes to maximize his overall expectation	Player wishes to maximize his hourly expectation	Player wishes to maximize his sessional expectation
2	yes	no	yes
4	yes	no	yes
8	yes	yes	no
16	yes	yes	no

Example 39:



Dick believes that, if his opponent offers the cube at level 2 and he accepts, his expectation is about - 0.9 of a point. This expectation is calculated taking into account the strength of his opponent. (If the reader disagrees with the statement that the expectation is about - 0.9 of a point, he could move one or two men until he reaches a position which will enable him to evaluate Dick's expectation at about - 0.9 of a point.) The opponent offers the cube at the following levels: 2, 4, 8, 16.

A) Dick wishes to maximize his overall expectation. Should he accept the cube?

Dick figures that, when the cube is offered at level 2, he will lose 0.9 of a point, if he accepts, as opposed to losing 1.0 point, if he refuses. Dick must, therefore, accept the cube whether it is offered at levels 2, 4, 8 or 16.

B) Dick would like to maximize his hourly expectation. Knowing that the past (and expected) performances are worth five points an hour, should he accept the cube?

Dick must compare his hourly expectation, if he accepts, with his hourly expectation, if he refuses. If Dick refuses, he wins five points an hour playing with the opponent. If Dick accepts this cube at level 2, he has a net gain of 0.1 point per game. Supposing that this position takes five minutes to play out, there would be 12 games an hour and Dick would then make 1.2 points an hour, playing this position as a proposition. He should, therefore, pass and move on to the next game.

If Dick accepts this cube at level 4, his hourly expectation becomes 2.4 points an hour. He should again pass and move to the next game because, if he refuses, his hourly expectation is five points an hour. If Dick accepts the cube at level 8, his hourly expectation becomes 4.8 points an hour. If he refuses, his hourly expectation is five points an hour. The cube is marginally acceptable. If Dick chooses to play this position by accepting the cube at level 16, then he will make about 9.6 points an hour. Dick should accept, because, if he refuses, he wins only five points an hour.

C) Dick is 10 points ahead in this session, with 10 to 15 minutes left to play. Dick wishes to maximize his expectation for the session. Should he accept the cube?

Peter is of the opinion that the cube should be accepted at level 2, that the cube is marginally acceptable at level 4, and that it should be refused at levels 8 or 16.

The summary for the position just analyzed is as follows:

Should a player accept the cube?

Level of the cube offered	**Player wishes to maximize his overall expectation**	**Player wishes to maximize his hourly expectation**	**Player wishes to maximize his sessional expectation**
2	yes	no	yes
4	yes	no	marginal
8	yes	marginal	no
16	yes	yes	no

* * * * *

This chapter has offered certain principles and some specific examples. Peter acknowledges that many experts would disagree with some analyses, but he maintains that **conventional doubling theory must take into account certain principles of money management.**

Most experts are aware of the fact that it is very logical to play with the goal of maximizing the hourly expectation because **time is money.** A whole book could be devoted to this, yet virtually no material is presently available on this subject.

CHAPTER 9 - PRINCIPLES OF MONEY MANAGEMENT IN PRACTICE

Peter explains to John that, to win at backgammon, he must be better than his opponent, but to win money, he must be stronger than his opponent and he must correctly apply the principles of money management. The intent of this chapter is to succinctly enumerate these principles with the help of a few mathematical formulas.

Peter emphasizes that these principles, whether they be established in a theoretical fashion or not, are all based on experience. Consequently, an experienced gambler with no mathematical background, would likely give advice very similar to what follows in these pages.

The ideas presented thus far, and which are reviewed in Diagram 7, are theoretical in nature. Peter believes that any theory that cannot be put into practice is invalid. He will complete his explanation of money management by showing John how he should proceed step by step in applying the principles of good money management.

To begin with, John must establish his maximum bet, and this can be done even before he meets his opponent. John could use Formula 19 as described in this publication or he could use any other logical approach as long as it takes all the following variables into account:

- his long-term bankroll
- his single-session bankroll
- the amount of money that the opponent can afford to lose
- the probable number of games per session
- the cube factor (based on the rules pertaining to the cube)
- the potential number of opponents

To establish his long-term bankroll, it is strongly suggested that the player only risk an amount that he can afford to lose over the long run. Peter thinks that this amount should not exceed what a player could lose in a year without affecting his own standard of living. John's response is to say that he can afford to lose $2,000 in the coming year, but that it would be a great blow if he lost $3,000 or $4,000. John notes, therefore, that his long-term bankroll is $2,000.

Peter feels that the single-session bankroll should vary from 5% up to 15% of the long-term bankroll and has already noted that 10% appears very reasonable. On the basis of this suggestion, John values his single-session bankroll at 10% of $2000, i.e. $200. If John loses his single-session bankroll, he should normally choose to end the session in which he is engaged.

The amount risked in one session should neither exceed the one-session bankroll nor the amount of money the opponent can afford to lose. But for the moment, let's establish the maximum bet, supposing that the opponent has the money.

Peter reiterates that the minimum number of games played in one session is 20. The number of games to be played in a session depends, among other things, upon the "speed" (8 games/hour or 15 games/hour) of the player. John considers that, for himself, a normal session would be played for five hours at a rate of 12 games an hour. He thus concludes that the likely number of games per session should be around 60 (N_{prob} = 60 games).

To evaluate the cube factor, John must begin by making a list of his usual rules. John tells Peter which rules he uses:

- no automatic cube (in some cases, one allowed)
- Jacoby's rule is in effect
- the opening throw must be played

Based on these rules and the results of his past sessions, John values his cube factor at 2.0 (C_f = 2.0). If a rule changes, John may use Table 8 entitled "Example of Reasonable Cube Factors" to make an adjustment.

The evaluation of the maximum bet, when John plays with just one opponent (N_o = 1), is made using Formula 19 as follows:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob}} \times C_f \times N_o} = \frac{\$2000 \times 10\%}{\sqrt{60} \times 2.0 \times 1} = \$12.91 \text{ a point}$$

The aim of the above formula is to evaluate his maximum bet and the result should serve as a guideline. Even if the theoretical maximum bet is $12.91 a point, the practical maximum bet is $10 a point.

Now that John has established his maximum bet at $10 a point, he should not try to impress his friends by betting $20 a point. Similarly, he should not let himself be intimidated by the fact that his opponent could play at $50 a point. In other words, he must play according to his own means, not those of his neighbor.

The amount John can risk in a given session ($200) and the maximum bet ($10 a point) can be determined long before he enters into discussion with an eventual opponent. Determining which criterion a gambler wishes to play by can also be accomplished before meeting an opponent. As already explained on numerous occasions, the possibilities are as follows:

a) The player wishes to maximize his probability of success. He, therefore, plays the cube so as to maximize his expectation for each game;

b) The player wishes to achieve a pre-established probability of success. To do this, he must have recorded previous results against a given opponent, and must then apply Formula 20. The player then handles the cube so as to maximize the expectation of each game;

c) The player wants to play in such a fashion as to maximize his hourly expectation. He thus follows those strategies that maximize the hourly expectation. The "hourly expectation formula" suggests that the gambler should:

   1) play using his maximum bet if he is stronger than his opponent;

   2) try to maximize his expectation by concentrating more than his opponent and by showing better self-control;

   3) play more games per hour;

   4) modify his approach to the cube, so as to maximize his hourly expectation;

   5) try to play as long as possible.

John chooses to maximize his hourly expectation. Thus, before he meets his opponent, John has already determined the following:

1) his single-session bankroll is $200;
2) his maximum bet (based on a cube factor of 2.0) is $10 a point;
3) his criterion is the maximization of the hourly expectation.

Now John is ready to face an opponent. Because his pre-established maximum bet of $10 a point is established with the assumption that the opponent can afford to lose $200, his main concern is to determine how much the opponent can afford to lose. If the opponent can lose (and pay $200), the maximum bet remains $10 a point. If, on the other hand, John believes that the opponent can afford to lose only $100, his maximum bet will decrease from $10 a point to $5 a point.

John could meet different kinds of opponents, namely: unknown opponents or known opponents who could be weaker, of the same skill, or stronger. If he wishes to play against an unknown opponent, he should try to get information about him, as discussed in Section 7.2. If the opponent accepts John's rules, then John must determine the bet he will use. This bet must be determined by taking into account the evaluation of John's strength. If John believes that he is a black-belt backgammon player, then the likelihood of an unknown player being stronger than he is not very great; in this case, he could play with his maximum bet. If John thinks that he is a green or blue belt (advanced player) and believes that he has a 50% chance of winning against an unknown player, then he can choose a bet that represents 50% of his maximum bet. John believes that he belongs to this class. On the other hand, if John were to think of himself as a beginner (yellow belt) and as having about a 10 to 20% chance of winning against an unknown opponent, then he would use a small bet which could represent 10 to 20% of the maximum bet.

When facing a weaker opponent, John should try to play at $10 a point, as long as his rules concerning the cube are accepted. By playing at $10 a point, John will maximize his hourly expectation. If the opponent suggests a bet of $1, $2 or $5 a point, John may agree to play, but he should try to change the rules so as to have a cube factor greater than 2.0. If the opponent suggests playing with a bet lower than $1, John might refuse since it's not advisable that the ratio between the maximum bet and the minimum bet be greater than 10. By agreeing to play at a very low bet, he would probably not be motivated to play well. On the other hand, he might decide to use such an occasion to give the impression to his opponent (and to the kibitzers) of being a weak player by purposely making mistakes and losing, for example, 10 points. He should try to "size up" his opponent and force him to play a little higher than his maximum bet (as long as this amount does not exceed John's maximum bet). For example, if his opponent's maximum bet is $2, John should try to play at $5. John should try to keep a low profile, to "humor" his opponent, and to keep in mind that if he wins too much money in the same session, he might lose his opponent. He should try to convince his opponent that if he, John, wins, it's because he is lucky, and that if he loses, it's because his opponent is stronger.

If John chooses to play against an opponent he considers to be of the same skill, he should play from $1 to $10 a point as long as his rules about the cube are accepted.

Even if, in principle, John should not choose to play against a stronger opponent, he might consent, considering this to be a learning investment. John should not try to beat a well-known stronger opponent merely for the purpose of beating him. He should never forget to leave his "ego" at home, nor forget that his only motivation must be making money. He should be willing to make such an "investment" by playing against a stronger opponent, if and only if, he believes that this investment will be profitable for him later on. In other words, it is probably good to be willing to lose $100 against a stronger opponent if this investment can bring $200 or more in the near future. Obviously, John should try to play against a stronger opponent with his minimum bet. By doing so, he maximizes his hourly expectation or, in other words, he "minimizes his hourly losses".

Peter points out that if John chooses to play chouette, at four as an example, then he should divide his maximum bet by three. Peter emphasizes that there are advantages and disadvantages at playing in a chouette. For example, this game has more of a social character than two-man play. On the other hand, there is the sad, but real possibility of collusion between "opponents". Peter suggests that John play in chouettes only when the stakes are low.

Once John has chosen an opponent and has established rules and betting values, he must then determine two other elements: 1) How will payment be made? and 2) How can play be ended? It is strongly suggested that the method of payment be clearly established before the beginning of play; for example, payment could be made, in cash, after every five or 10 points, or after every 10 games. It is so common for players to wait until the end of a session to be paid, and then to receive nothing more than a "rubber" check, that it is necessary to decide upon a payment plan before beginning to play and to insist that the rules be kept. The fact that John can afford to play at $10 a point is no guarantee that his opponent can do likewise. If his opponent is unable to pay cash and if John chooses to continue to play anyway, then he proceeds at his own risk.

It is also highly recommended that the players define how to end the session. Even if, as a rule, the loser can stop any time, it is preferable that this rule be clearly indicated at the beginning of play. The two players may, for example, agree to play 30 or 40 games and then decide, once that point has been reached, whether or not to play another 10 games. They could also decide to play by the clock, say, for three hours, and then, upon agreement, choose to play for an extra hour. If the rules for ending play have not been established beforehand, then the losing player can insist on continuing. Assuming that John is winning, he could then be obliged to play much longer and much later than he had wanted to.

When all the following rules have been agreed upon:

- rules relative to the cube have been determined;
- the bet has been established;
- pay-off arrangements have been determined;
- rules have been established for ending play;

and before play actually begins, Peter notes that John should insist on playing with precision dice and dice cups. John accepts these bits of advice, but notes that they are not related to the principles of money management. Peter responds by noting that it is possible for a player's technical superiority to be offset by the use of dubious practices. For example, a player may have an expectation of +10% per game if precision dice and a dice cup are used, and an expectation of -10% per game if dice of dubious quality are used, and without the dice cup. Peter observes that John does not seem familiar with the techniques that some players may use in order to cheat with dice. It is Peter's suggestion that John buy the following book: Scarne, John, "Scarne On Dice", Eighth Revised edition, Crown Publishers Inc., and read chapters 11-14, entitled:

- Gamblers, Hustlers and Cheats;
- Crooked Dice: Inside Work;
- Crooked Dice: Outside Work;
- Crooks at Work: Moves.

John needs to be warned against playing on the opponent's ground. He desires further explanation. Peter replies: "Let's suppose that you come from town A (or club A) and you go on to town B (or club B) to meet an opponent. What might happen? Most of the time, your opponent's friends will come and look around.

Unless you keep an eye on them, you'll be unable to tell if they are giving signals to their friend. In other words, they might cheat. Furthermore, if an argument about the rules arises, you are almost sure to lose. The player has the advantage on his own ground. To avoid such a difficulty, you can suggest playing on neutral ground."

Once the game starts, John should record all results, even if the opponent does the same. John will compile them later in order to have up-to-date information on his opponent.

Once the game is under way, John has to keep in mind the following: to play with unbroken concentration; to maintain self-control; to be patient. These tips will help John to put the odds on his side in his attempt to maximize his hourly expectation. This edge will often show only toward the end of the session. If John believes that, for one reason or another, he is losing his powers of concentration or self-control, he is justified in ending the session in which he's engaged. Peter points out to John that players who constantly complain about the bad dice they receive are more apt to lose their self-control. Consequently, he advises John never to criticize or make remarks about the dice, so as to keep his self-control. He also points out that if casinos offer free drinks to customers, it's not out of kindness but with a view to making more money. As a general rule, players who use alcohol have a tendency to show less concentration, to lose self-control, and to be less patient. It is, therefore, not necessary to lose one's single-session bankroll in order to end a session. If a player's concentration is affected by fatigue, alcohol, loud music or anything else (e.g. drugs), it is then desirable to end the session. Peter points out to John that, if he wants to play backgammon on a professional level, he must achieve a high level of discipline.

If John is losing, he should not ask his opponent to increase the bet. On the other hand, if John is winning, he is theoretically justified in accepting this offer if he judges that his chances of winning each game are much greater than what they were at the beginning of the session. All the same, John should only agree to increase the bet gradually. For example, he could accept going from $5 a point to $10 a point, but should refuse going from $5 to $20 a point. In practice, John may be justified in refusing to increase the bet if he thinks that his opponent will "steam" more and that he can take advantage of the situation.

If the losing player offers John to play at double or nothing (no cube, no gammons), he should evaluate the overall situation. Often, the player making this offer has not, as yet, paid up and it is possible that he'll have a certain amount of trouble paying what he owes. John would thus be well-advised to refuse such an offer and to insist on being paid. Although Peter would rarely accept such a proposition himself, he emphasizes to John that it is entirely possible that the losing player may not have any difficulty in paying whatever the amount that might be due. In such a case, the winning player could, theoretically, be justified in accepting such a proposition if he believes that his chances of winning are over 50%. But, in practice, it doesn't seem logical to risk in a single game the total amount won after playing several hours. A player may, therefore, be perfectly justified in rejecting such an offer.

Toward the end of the session, Peter suggests that John handle the cube to maximize his sessional expectation. John could also suggest and accept settlement with the goal of maximizing his sessional expectation.

John must be disciplined enough to quit if he has lost his single-session bankroll, because it becomes very difficult to avoid losing control of one's emotions in such a situation. John must have the wisdom and the discipline to end the session. This may be easy to say but difficult to put into practice for the very reason that when a player loses, he is frustrated. If, on the one hand, John must put a limit on the amount he can lose, he should not, on the other hand, limit the amount he can win.

When the session is over, John saves the scoresheet which he has filled out and takes it home to keep the file on his opponent up-to-date. Example 12 shows how to compile this information. For example, John might keep a card on file (3 1/2" by 5") for each opponent. If the file is up-to-date, then he can easily establish, in an objective way, his order of preference for meeting opponents. John could also indicate on this file his evaluation on how much each opponent can lose.

Peter suggests that John might open a special bank account in which he can deposit (or withdraw) weekly the amount won (or lost). This procedure will allow him to know exactly what his real current situation is concerning his real winnings and losses. John accepts this advice, even if he believes that it has nothing to do with money management.

Before facing his next opponent, John will have to adjust his long-term bankroll, taking into account the amount he has just won or lost. If he has lost $100 then his long-term bankroll becomes $1,900; if he has won $200, then it now stands at $2,200. Next, John will re-evaluate the amount of his maximum bet, taking his new long-term bankroll into account. Since Formula 19 gives a theoretical maximum bet, and since its results serve as a guideline for establishing the practical maximum bet, it follows that a player who is winning a little bit or losing a little bit need not necessarily change his bet. Based on the variables that John has established for himself (one-sesson bankroll = 10% of long-term bankroll; probable number of games in one session = 60; cube factor = 2.0, number of opponents = 1), the ratio between his long-term bankroll ($2000) and his theoretical maximum bet ($12.91) is 155. In other words, the maximum bet for John should be approximately equal to the long-term bankroll divided by 155. Based on this last consideration, Peter suggests that John apply the following maximum bets:

Long term bankroll including cumulative winnings and losses (amounts given as examples only)	**Maximum bet for one session (with the above assumptions)**
$100 to $250	$1
$250 to $600	$2
$600 to $1,300	$5
$1,300 to $2,500	$10
$2,500 to $5,000	$20
$5,000 to $7,000	$40
$7,000 to $10,000	$50

Peter believes that John must play according to the maximum bets established above because, in doing so, John will be using the strategy he considers to be the safest and the most efficient. But if John loses, say 75% of his long-term bankroll in a row, he should consider giving-up backgammon and turn to another game like chess or bridge.

John is very satisfied at having met Peter because he now knows and understands everything he wanted to know about the principles of money management as applied to backgammon. Peter points out that, even if John has acquired a good theoretical background on the subject, he needs a good deal of effort, patience and perseverance to put his knowledge into practice. Theoretical knowledge not put into practice has no practical result. Peter points out that the real backgammon winner wins money; in other words: **"YOU CAN ALWAYS TELL A HUNTER BY HIS HIDES".** One of the best ways of "catching good hides" is to apply good principles of money management at all times.

Now that John has been able to establish his own appropriate bet based on his own variables and his own criteria, he would like to know how Peter proceeds in evaluating <u>his</u> appropriate bet. To satisfy John's curiosity, Peter explains his personal approach by giving the following example.

Example 40: (Peter's variables and criteria)

First, Peter estimates his long-term bankroll at about $3000; this bankroll is subject to the variations of cumulative winnings and losses. His single-session bankroll is obtained by using 10% of his long-term bankroll, that is to say, he is presently at approximately $300 (amount risked = $300). Peter guesses that a normal session should have between 70 and 80 games (N_{prob} = 75 games). Peter applies rules concerning the cube in such a way as to have a cube factor of about 2.0 (C_f = 2.0). When there is only one opponent (N_o = 1), the maximum bet is established by using formula 19 as follows:

$$\text{Maximum bet} = \frac{\text{Amount risked}}{\sqrt{N_{prob} \times C_f \times N_o}} = \frac{\$300}{\sqrt{75 \times 2.0 \times 1}} = \$\,17.32$$

The theoretical maximum bet is $17.32 and the practical maximum bet is $20 a point. Peter often plays at $5 and $10 a point. Because it's not common to play at $15 a point in practice (either $10 or $20), he occasionally plays at $20 a point. He never plays with a bet of over $20, but he believes that eventually it will be possible. Occasionally, when he plays at $1 a point, he often loses interest because the bet is too low.

As far as the criterion used, Peter always tries to establish his bet in such a way as to obtain a probability of success of 80 to 90%. For example, when his single-trial probability of success (based on a large number of games) is evaluated at 54%, the appropriate bet is evaluated by using formula 20, as follows:

$$\text{Appropriate bet} = \frac{\text{Amount risked} \quad \times \quad \text{Log}(Q/P)}{C_f \times N_o \quad \times \quad \text{Log}(1\text{-}P_{success})}$$

To obtain a probability of success of 80% the appropriate theoretical bet is $14.94 a point and to obtain a probability of success of 90%, the appropriate theoretical bet is $10.45 a point. If Peter knows that the opponent does not increase the bet when the latter is losing, then the bet is fixed at $10 a point. On the other hand, if he knows that his opponent increases the bet even if losing, then he starts with a bet of $5 a point. To be completely explicit, he has prepared the following table:

Single-trial probability of success (P)	**Appropriate bet to obtain a probability of success of 80%**	**Appropriate bet to obtain a probability of success of 90%**	**Practical bet if the opponent does not increase bet when losing**	**Practical bet if the opponent increases his bet when losing**
40 to 50%	nil	nil	$1 if possible	$1 if possible
52%	$ 7.46	$ 5.21	$ 5	$ 2
54%	$ 14.94	$ 10.45	$ 10	$ 5
56%	$ 22.48	$ 15.71	$ 20	$ 5 or $ 10
58%	$ 30.08	$ 21.03	$ 25	$ 10
60%	$ 37.79	$ 26.41	$ 25	$ 10

Since Peter plays to have a pre-established probability of success, he handles the cube in the conventional way, that is to say, to maximize the expectation of each game.

When Peter plays against an unknown opponent, he is unable to use the preceding criterion (because he has no information and is not able to calculate the single-trial probability of success), and most of the time he plays at $5 a point if his rules about the cube are accepted.

If toward the end of a session, he has a reasonably positive score, then he plays to maximize his sessional expectation (i.e. to keep most of his winnings).

In a sense, Peter tries to follow as closely as possible the advice he has previously given to John, but he utilizes his own variables and criteria.

* * * * *

To be absolutely certain that John has mastered the principles explained so far, Peter proposes "game situations" involving fictitious characters and facts derived from real-life situations. The question for John is to decide whether or not the player, in the examples given, has followed the rules of money management.

Example 41:

Player A bets by dividing his pocket money by 50, and considers this to be in accordance with the principles of money management. Having $250 in his pocket, he is then willing to play at $5 a point against anybody. Has this player respected the principles of money management as outlined in this publication?

1) Establishing the maximum bet by dividing the single-session bankroll by 50 does not seem unreasonable. However, A should be able to determine this division factor himself, taking into account his long-term bankroll, the number of games to play, the rules relating to the cube, and the possible number of opponents. The fact that A does not know how the number "50" is arrived at constitutes a weakness.

2) A does not seem to make any distinction between his single-session bankroll and his pocket money. It is also within the realm of possibility that such a player might not make a clear distinction between his single-session bankroll and his long-term bankroll. If the money that A carries on him (i.e. $250) represents the value of his paycheck and if A has a long-term bankroll of $1000, then his single-session bankroll should range from $50 to a maximum of $150. The single-session bankroll should not be equal to the pocket money.

3) The player seems to be willing to play against any opponent without evaluating the other's strength beforehand. This, in itself, could be very destructive.

4) The player obviously makes no distinction between maximizing his probability of success and maximizing his hourly expectation. Choosing one of these criteria as opposed to the other, will give totally different results. For example, a player who goes up against a weaker opponent, must use his maximum bet if he wishes to maximize his hourly expectation and must use the minimum bet if he wishes to maximize his probability of success. The appropriate bet is not necessarily the maximum bet.

5) Even if A believes that he is using a proper system of money management because he can divide, in his head, the amount of money in his pocket by 50, for all intents and purposes, he has completely ignored the basic principles of money management as they apply to games in general and as they apply specifically to backgammon.

   Note: John points out to Peter that, before meeting him, he used the approach outlined in this example. Peter answers that he began with this example purposely, in order to show John the progress he had made.

Example 42:

B has a long-term bankroll of $1,000 and he establishes the amount of his bet by dividing this bankroll by 100. B can thus afford to play at $10 a point. He meets a new opponent and agrees to play against him at $10 a point. Shortly thereafter, he agrees to have no limit placed on automatic cubes. Furthermore, he accepts the rule allowing the player the option of refusing the opening throw and to turn the cube to the next level, a new rule for B. After five or six hours of play, B has lost $500 and paid off his debt. Has he respected the principles of money management as outlined in this publication?

1) All remarks made in the previous example are also valid in this case.

2) B established the playing rules after agreeing upon the amount to be bet. The rules should have come first.

3) B should have limited the number of automatic cubes to one or two.

4) B should not have accepted playing with the rule that allows a player to refuse the opening move and to turn the cube to the next level. The reason for this is that his bet does not take this rule into account. B should either have refused to play by this new rule, or he should have lowered his bet.

5) B was playing against an unknown opponent. In such a case, he should have tried to evaluate the other's strength by asking him some questions. He should also have tried to evaluate how much the opponent could afford to lose. B should not have played with his maximum bet of $10.

6) B should have quit when he had lost about 10% of his long-term bankroll, i.e. about $100.

7) Altogether, B made a large number of errors. Still, experience involves learning from one's mistakes.

Example 43:

C, an average player who has a long-term bankroll of $1,000 and who is playing with a maximum bet of $5 a point, wins a tournament and $500. Soon after, C increases his bet to $10 a point and, after winning $1,000 with this bet, is prepared to play at $20, and even $50 a point. One evening, playing at $20 a point, he wins $1,000. All is well, C now has a long-term bankroll of about $5,000 and looks for opponents at $200 a point. Is C applying the principles of money management as explained in this publication?

1) Remarks made concerning previous examples are valid here. C probably doesn't know the basic principles of money management.

2) At the beginning of his career, C divided his long-term bankroll by 200 to determine his maximum bet ($1000/200 = $5). This factor now becomes 25 ($5000/25 = $200). Why has this change been made? C probably would not be able to answer this question except to say that he's "hot" and that he's taking his chances while things are working well.

3) This player has not yet faced a stronger opponent and he is pushing his luck. He's free to do so, but he is defying the basic principles of money management which suggest playing prudently and limiting the one-session bankroll to 5% to 15% of the long-term bankroll. By playing at $200 a point C could easily lose 25 points in a session which represents his long-term bankroll. All that can be done is to wish good luck to those who defy the most elementary rules of caution.

Note: Peter knows some players who bravely defied the basic rules of money management. Many played for up to $100 a point a few years ago but now restrict themselves to $5 a point. The experience gained by watching these players teaches us that the gambler who faithfully respects the basic principles of money management has a better chance of a promising career. John notes that he too has known players who have had the same experience.

Example 44:

D is a white-collar worker who earns $30,000 a year and who figures that he has a $5,000 long-term bankroll. He sets the amount he can afford to lose in one session at $500. In an exclusive club, D meets, for the first time, a businessman who sports a diamond ring and looks rather prosperous. The rules of play are established and they agree to play at $20 a point. After two hours of play, D is 20 points ahead; he then indicates that he would like to quit in a few minutes. His opponent suggests that they play at double or nothing (no cube, no gammon); D accepts and he wins. He thus has a lead of 40 points or $800. His opponent again offers a chance to go double or nothing and D again accepts and wins. The opponent is willing to try a third game at double or nothing but D refuses. His adversary is thus obliged to make out a check for $1,600. A week later, D discovers that the check has bounced. Has he respected the principles of money management?

1) Even if D has not determined his maximum bet based on a mathematical formula, a maximum bet of $20 a point seems reasonable for a player who has a long-term bankroll of $5,000 and a single-session bankroll of $500. All the same, this bet is reasonable if, and only if, the cube factor is around 2.0.

2) D has used his maximum bet playing against a stranger. His opponent likely would have accepted playing at $10 a point and in doing so, D would have had a chance to analyze him. It is suggested not to play the maximum bet against a total stranger. The appropriate bet is not necessarily the maximum bet. D does not appear to have made this last distinction.

3) The fact that a certain player can gamble $20 a point does not mean that his opponent can afford to do the same. D was too trusting. He should have insisted on being paid after every $100 won, or after every five or 10 points. The method of payment should have been defined at the beginning of the session.

4) When one player offers to play at double or nothing, it often indicates that he may have trouble paying off. D may have had a 50 to 75% chance of being paid $400, but the likelihood of receiving $1600 was practically nil. D should have refused to play double or nothing and, instead, insisted on being paid what was already owed.

5) D has begun to collect rubber checks. He most likely has friends who "own" even more impressive collections. If D does not make the same mistake twice, then the lesson he has learned will not have been that expensive.

Example 45:

E has a long-term bankroll of $1,000, a one-session bankroll of $150. He believes that his maximum bet is $10 a point. He plays against a stranger, but has enough sense to play at only $2 a point. After 2 hours of play, E has won 20 points and has collected his $40. One week later, he meets this same opponent. They clearly establish the following rules:

- no automatic cube
- Jacoby's rule is in effect
- the opening move has to be played
- bet of $10 a point
- payment after every $50 lost

E agrees to play at $10 a point because of the strength of his past performance. After 5 or 6 hours, E has been losing regularly and has already paid off $300. Has he been faithful to the principles of money management?

1) First of all, with the above rule, the maximum bet of $10 a point, seems too high.

2) A player cannot deduce that he is superior to an opponent after only 20 or 25 games (or 2 hours of play). The opponent may be playing it coy. E was too trusting.

3) The $10 bet could have been acceptable if E really were superior, but it would have been wiser to increase the stakes more gradually. A $5 bet would have been more sensible.

4) E should have stopped playing once he had lost his one-session bankroll of $150.

Example 46:

F has a $5,000 long-term bankroll and he can risk $500 in a single-session. He plays against an opponent he knows. The rules are established:

- . no automatic double
- . Jacoby's rule is in effect
- . the opening move has to be played
- . bet of $10 a point
- . payment every 10 points

After 5 or 6 hours of play, F, who is truly superior to his opponent (during that period of time), has taken in $400. His opponent insists on having a chance to get even. Since he doesn't have to work the next day, F gives him the chance to do so. Two or three hours later, F is no further ahead than he was before. Even though he is beginning to be tired and a little bit drunk, F continues to play at his opponent's insistence. F agrees to increase the bet at $20 a point and after another 2 or 3 hours of plays, F has lost everything he had won and an additional $1,000. Has F made a mistake in his application of money management principles?

1) With the above rule, the maximum bet of $10 a point looks very reasonable.

2) F did not decide, at the outset, how the session could be ended.

3) F should not have played more than 5-6 hours because after that point, he obviously became inferior to his opponent.

4) F should not have accepted playing at $20 a point because, at that time, he was inferior to his opponent. F should definitely play to maximize his sessional expectation and insist on postponing play for the next day or however long might be necessary.

5) F should have stopped after having lost his one-session bankroll of $500.

Example 47:

G, who has a single-session bankroll of $200, plays often against the same opponent and keeps records of each of their meetings. After 10 very representative sessions, G has made $500. G is used to playing at $5 a point. The usual rules are established:

- no automatic double
- Jacoby's rule is in effect
- the opening move has to be played
- play at $5 a point
- payment every 10 points
- stoppage of play on 10 games notice

After 3 hours of play, G was 10 points ahead, and his opponent gave notice that he would quit after 10 more games. During these 10 games, G lost 18 points and paid-up. Has G made a mistake?

1) G has not been playing against a stranger. The rules have been clearly established, the bet is acceptable, and the rules for payment are also clear.

2) The fact that G lost 18 points in 10 games indicates clearly that he has not tried to maximize his sessional expectation. G has not made any mistakes with respect to the rules of money management, but he might have opted for the maximization of his sessional expectation.

Example 48:

H has been playing backgammon for six to eight years, has read at least ten books on the subject, and is considered a strong player in his circle of friends. H values his long-term bankroll at $2,000 and can afford to lose $200 to $300 in the course of a single-session. H plays at $10 a point, which is his maximum bet. Since H has scrupulously analyzed all the examples about how to handle the cube, several of his friends argue, with reason, that he plays the cube according to the book (i.e. he plays to maximize the expectation for each game). For example, once he played a backgame because he estimated that he would only lose, on the average, 0.9 of a point by accepting, whereas by refusing he would lose 1.0 point. H does not take into account the time necessary to play that game. Does H respect the principles of money management as put forward in this publication?

First of all, H seems to be playing with a reasonable bet if the cube factor is around 2.0. By playing at his maximum bet of $10 a point, H thus agrees to play according to the criterion known as "maximization of the hourly expectation". In fact, if H wishes to maximize his probability of success, then he should use the smallest bet possible.

In determining his bet so as to maximize his hourly expection, H should also, in order to be consistent, handle the cube in such a way as to help maximize his hourly expectation. Until now, books on the subject have only defined the criteria to be used to maximize the expectation for each game as opposed to maximizing the hourly expectation. Since H respects the principles outlined in the books, it follows that he plays the cube so as to maximize the expectation of each game.

H is faced with a contradiction that he has probably not noticed. He gives primary importance to the element of time when he plays with his maximum bet and assigns a role of secondary importance to time in playing the cube so as to maximize the expectation of each game. The fact that H plays with his maximum bet implies that he should also handle the cube so as to maximize his hourly expectation.

Example 49:

A player reputed to be an expert in backgammon plays with a bet which he considers high ($100 a point). His opponent is rather strong. They agree that, in order to quit, a notice of two games is sufficient. After one hour of play, the expert who has won 8 points, gives that notice. The day after, he tells all his friends that his own principles of money management state that a player should quit while winning. Has this player followed the principles of money management as outlined in this publication?

The fact that a player has to limit his losses doesn't mean that he must limit his winnings. If the player has correctly established his bet, then there is no obvious reason why a player should quit a favorable session. If this expert is stronger than his opponent, why does he quit so early? The expert would probably not be able to answer that question except to say that he didn't want to "kill his pigeon".

One basic principle of money management states that a player should not quit a favorable game. These principles indicate, among other things, that a player should play with his minimum bet if he wants to maximize his probability of success, and with his maximum bet if he wants to maximize his hourly expectation. When a player plays a favorable game, the only question to answer is: "What is the appropriate bet?" A player playing with an appropriate bet has no reason to leave a favorable game.

When the stronger player chooses to play with the criterion of maximizing his hourly expectation, he should not only play with his maximum bet, but also try to play as long as possible. A notice of 10 games would be more appropriate.

Therefore, this player may be a backgammon expert, but he can improve his own principles of money management by making a self analysis to determine whether or not he is really playing with his appropriate bet. The variables and criteria outlined in this book may help him.

CONCLUSION

Now the reader should be able to evaluate by himself his own variables and to choose his own criterion (or criteria) in such a way as to be able to establish his maximum bet by himself, particularly his appropriate bet.

The reader should also be able to explain, if necessary, the nuances included in the following notions:

- long-term bankroll versus single-session bankroll;
- maximum bet versus appropriate bet;
- determination of the bet to maximize the probability of success versus the hourly expectation;
- how to handle the cube to maximize the overall expectation versus the hourly expectation versus the sessional expectation.

If the reader has gained information which will enable him to make more money with backgammon, then the principal goal of this book will be attained.

As a final word of advice, it is to be reiterated that the true backgammon winner is determined by his winning, or, in other words, "YOU CAN ALWAYS TELL A HUNTER BY HIS HIDES", and one of the best ways of "CATCHING A HIDE" is to apply good principles of money management at all times.

ABOUT THE AUTHOR

Michelin Chabot graduated from Sherbrooke University in the Province of Quebec (Canada) as a civil engineer in 1972. He played chess seriously from 1964 to 1976, but gave it up to devote himself entirely to backgammon.

His first publication, "Backgammon - How Much Should You Bet?", is the only comprehensive guide to establishing an appropriate bet. The theories are interspersed with clear, practical examples to illustrate the principles of wise money management. The book breaks new ground, and is therefore a much needed, long awaited manual in the world of backgammon.

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