How to calculate exact game winning chances during bearoff

Bearoff positions without contact imply many cube decisions. Not only the moneygame takepoint is of interest but also an exact calculation of game winning chances (GWC) would be nice for use in matches.

The best known instrument is the effective pipcount (EPC). For some positions the EPC can be calculated easily over the board but for others it must be estimated by reference patterns and experience. In the first part we introduce a simple formula to estimate the EPC for positions up to 8 checkers. In the second part we show how the EPC difference influences the GWC.

Calculating the effective pipcount

The effective pipcount consists of the real pipcount and some wastage generated by ineffective use of dice rolls.

EPC = pipcount + wastage **(1)**

The EPC can be calculated multiplying the average number of rolls needed to bear off with the average number of pips within a single roll.

EPC = average number of rolls * 8.16 **(2)**

Walter Trice found the exact solution for n-roll positions. This means positions with all checkers on the acepoint.

EPC = n * 7 + 1 **(3)**

He also describes how to estimate the wastage for general positions. Positions with perfect distribution have about 7.5 pips wastage somewhat depending on the number of checkers. For positions between these extremes you have to rely on reference positions and experience.

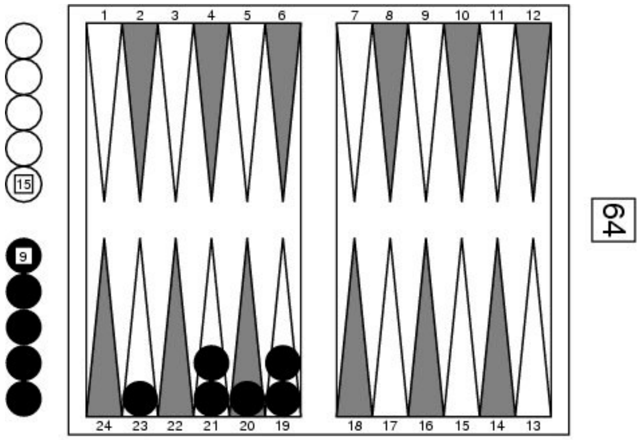
We use a different approach to calculate the EPC. All positions with up to 8 checkers within the homefield and the wastage of each position were read from the GnuBG (www.gnubg.org) bearoff database using bearoffdump.exe. This data was fed into a free program for linear regression (www.numericalmathematics.com). A six-fold linear regression was done to achieve coefficients for each point in the homefield. These coefficients have to be multiplied with the number of checkers on each point. The sum of the products plus a fixed offset gives the wastage. Two correction terms are described within the examples. For practical purpose we will use some slightly rounded coefficients. Exact results are given in the appendix.

Wastage = a0 + a1*Ch(1) + a2*Ch(2) + a3*Ch(3) + a4*Ch(4) + a5*Ch(5) + a6*Ch(6) **(4)**

with Ch(x): number of checkers on point x

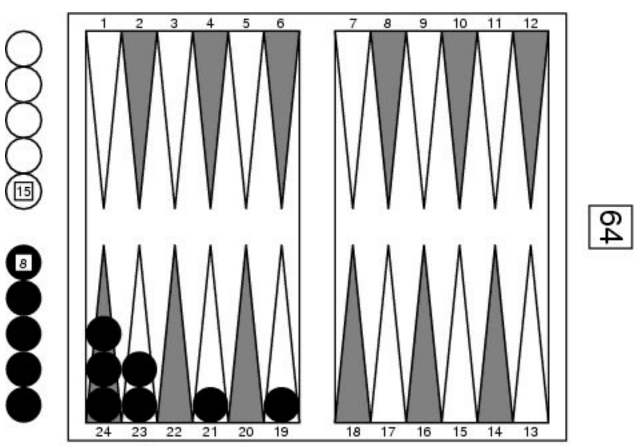
a0 = 4.5
a1 = 2.0
a2 = 1.3
a3 = 0.8
a4 = 0.4
a5 = 0.2
a6 = 0.1 **(5)**

Now we give three examples on how to calculate the wastage.



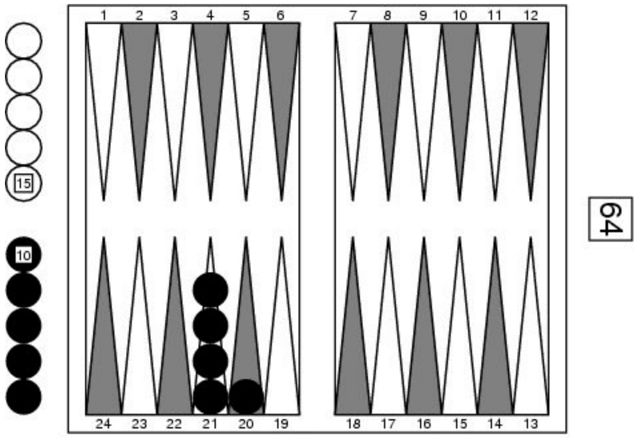
Pips = 27
Wastage = 4.5 + (2.0 * 0) + (1.3 * 1) + (0.8 * 0) + (0.4 * 2) + (0.2 * 1) + (0.1 * 2) = 7.0
Calculated EPC = 34.0
Database EPC = 33.5

Example 2 introduces the first correction term. **If there is at least 1 checker on the acepoint and at least 4 points are made, subtract 1 pip.**



Pips = 17
Wastage = 4.5 + (2.0 * 3) + (1.3 * 2) + (0.8 * 0) + (0.4 * 1) + (0.2 * 0) + (0.1 * 1) **- 1** =12.6
Calculated EPC = 29.6
Database EPC = 30.0

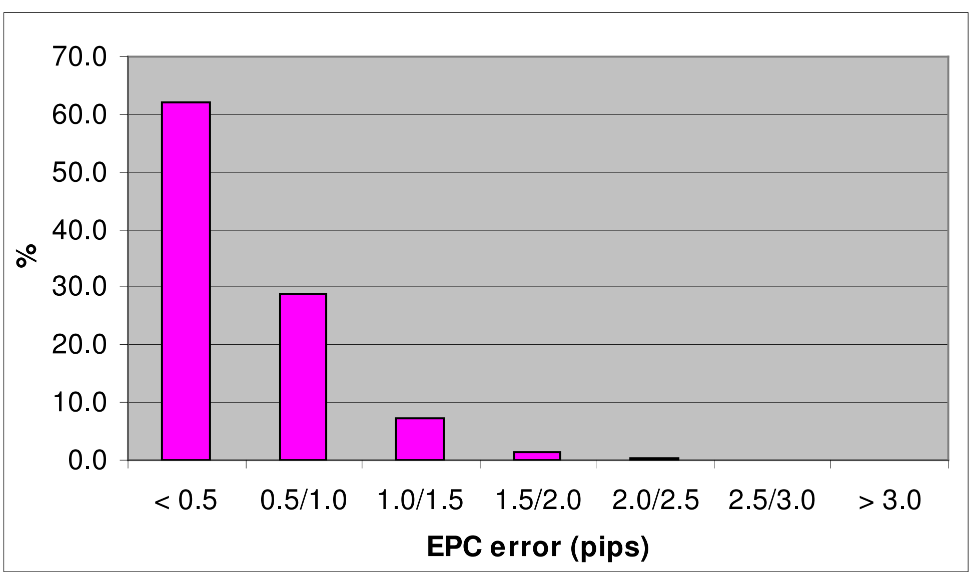
Example 3 introduces another correction term. **If there is a stack with at least 4 checkers, add 1 pip. If there is a stack with at least 6 checkers, add 1 more pip.**



Pips = 21
Wastage = 4.5 + (2.0 * 0) + (1.3 * 0) + (0.8 * 0) + (0.4 * 4) + (0.2 * 1) + (0.1 * 0) **+ 1** = 7.3
Calculated EPC = 28.3
Database EPC = 28.6

All 3002 positions with up to 8 checkers were tested. Average error per estimate is 0.48 pips.

Error (pips)	%	#
< 0.5	61.9	1857
0.5 < x < 1.0	28.7	862
1.0 < x < 1.5	7.4	223
1.5 < x < 2.0	1.5	44
2.0 < x < 2.5	0.4	13
2.5 < x < 3.0	0.1	3
> 3.0	0	0



Calculating game winning chances

After having estimated the EPC of a position you may ask how to use these results to calculate the game winning chances. To get an answer we conducted another experiment.

We compared GWC between several bearoff positions. Two classes of positions were chosen, pip positions and roll positions. Pip positions have a perfect distribution mostly on the 4, 5 and 6 point, sometimes there is one checker on 3, 2 or 1 point. The pip positions range from 8.2 to 100.2 EPC. Roll positions have all their checkers on the acepoint except of 1 chequer at most on another point. The roll positions range from 8.2 to 57.0 EPC because no roll position above 57.0 EPC can be created with a maximum of 15 checkers. The distance between each sample position within one class was chosen at about 3 pips.

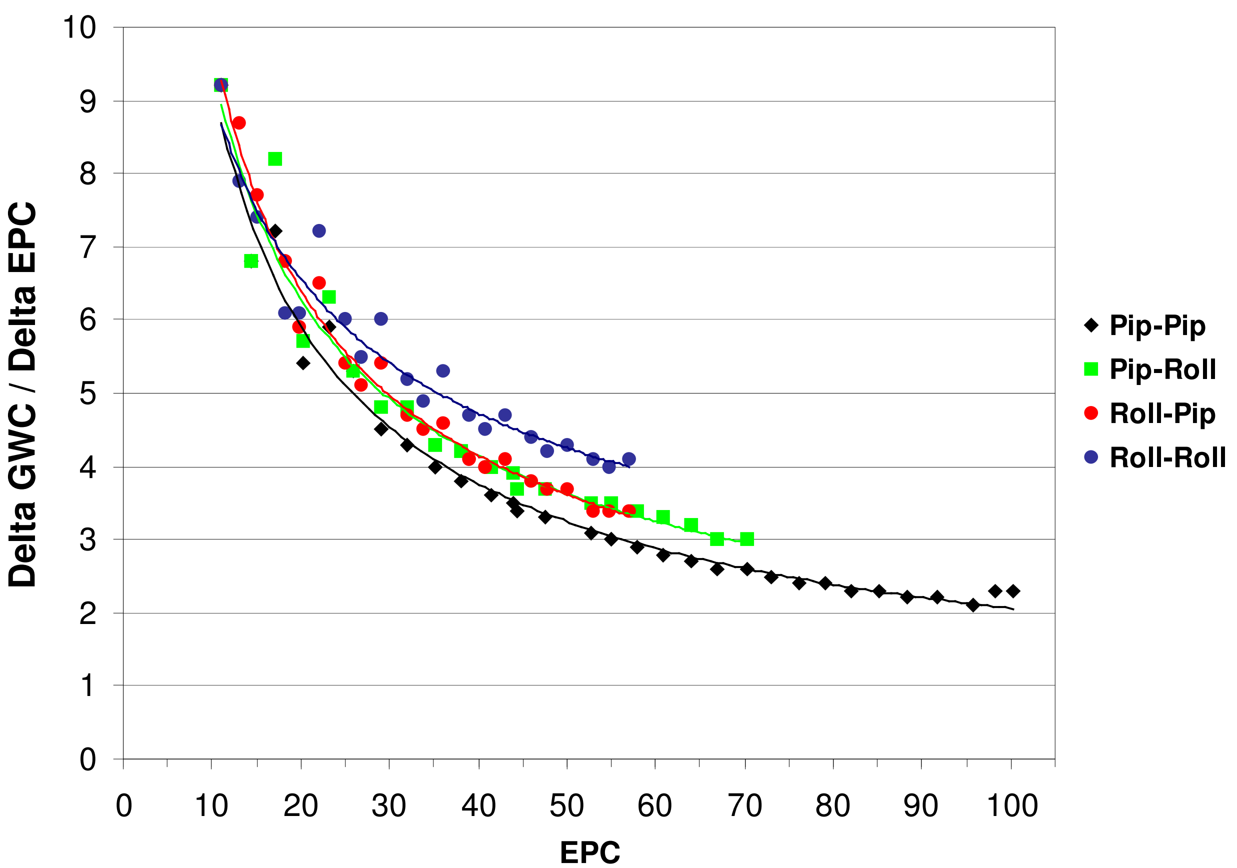
We calculated a value of GWC % per pip EPC.

$$\Delta GWC / \Delta EPC = (GWC - 50\ \%) / (EPC2 - EPC1 + 4.1) \qquad (6)$$

with

EPC1	EPC player 1
EPC2	EPC player 2
4.1	Pip correction for player 1 for being on roll

The diagram shows how much GWC % a player gains from 1 pip.This value depends on the EPC of the player being on roll and the kind of positions involved. It can be seen that one pip is much more valuable within short races than within long ones.



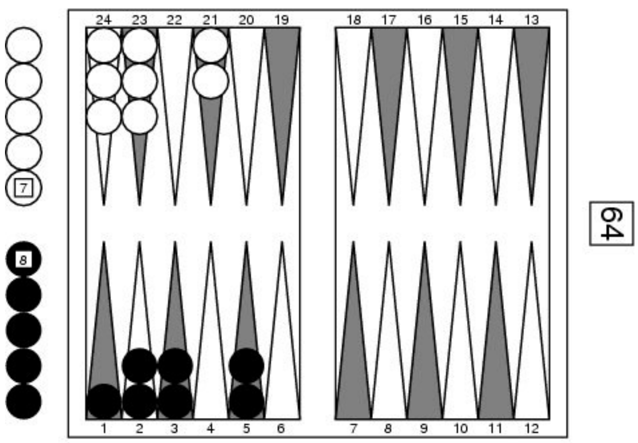
Following results for pip against pip positions were calculated from the formula of the trend line. Exact results can be viewed in appendix.

EPC1	ΔGWC / ΔEPC (%/pips)
105	2.0
75	2.5
55	3.0
45	3.5
36	4.0
30	4.5
26	5.0
22.5	5.5
20	6.0

(7)

Percentages for roll-pip and pip-roll positions are roughly 0.5 % above pip-pip positions.
Percentages for roll-roll positions are roughly 1.0 % above pip-pip positions.

Here is one example.



Player black (on roll)

Pips = 21
Wastage = 4.5 + (2.0 * 1) + (1.3 * 2) + (0.8 * 2) + (0.4 * 0) + (0.2 * 2) + (0.1 * 0) - 1 = 10.1
Calculated EPC = 31.1
Database EPC = 31.5

Player white

Pips = 17
Wastage = 4.5 + (2.0 * 3) + (1.3 * 3) + (0.8 * 0) + (0.4 * 2) + (0.2 * 0) + (0.1 * 0) = 15.2
Calculated EPC = 32.2
Database EPC = 32.4

(6) gives

GWC = 50 % + (ΔGWC / ΔEPC) * (EPC2 - EPC1 + 4.1) **(8)**

A quick look at (7) gives 4.5 % for pip-pip positions. But both sides seem to be between pip and roll positions. Thus we add 0.5 % to achieve 5.0 % as coefficient. Finally we get 75 % from (8). Database says 76.1 %.

Summary

EPC calculation

1. EPC = pipcount + wastage
2. Wastage = a0 + a1*Ch(1) + a2*Ch(2) + a3*Ch(3) + a4*Ch(4) + a5*Ch(5) + a6*Ch(6)
3. a0 = 4.5
   a1 = 2.0
   a2 = 1.3
   a3 = 0.8
   a4 = 0.4
   a5 = 0.2
   a6 = 0.1
4. If there is at least 1 checker on the acepoint and at least 4 points are made, subtract 1 pip.
5. If there is a stack with at least 4 checkers, add 1 pip. If there is a stack with at least 6 checkers, add 1 more pip.

GWC calculation

1. Calculate EPC for players on roll (EPC1).
2. Calculate EPC for player not on roll (EPC2).
3. Choose coefficient for pip-pip positions from table

EPC1	ΔGWC / ΔEPC (%/pips)
105	2.0
75	2.5
55	3.0
45	3.5
36	4.0
30	4.5
26	5.0
22.5	5.5
20	6.0

4. For pip-roll or roll-pip positions add 0.5 % to coefficient.
5. For roll-roll positions add 1.0 % to coefficient.
6. Between pip and roll positions do some interpolation.
7. GWC = 50 % + coefficient * (EPC2 - EPC1 + 4.1)

Conclusion

A linear formula for calculating the EPC for bearoff positions up to 8 checkers is introduced. Two correction terms give an improved accuracy for the estimated EPC. The average error of the estimate is 0.48 pips. The highest deviation from database values occur for some n-roll positions. **This error can be avoided by using Walter Trice´s formula for n-roll positions and Douglas Zare´s extension for near-to-n-roll positions (published on www.gammonvillage.com).**

Game winning chances can be calculated from EPC differences. The value of EPC pips is described by percentage coefficients that depend on the EPC of the player being on roll. These coefficients need some correction depending on the category of the positions (pip, roll).

The GWC estimate error is usually in a range of +-2 %. The calculation of GWC is valid down to 20 pips EPC. At this point winning chances start to fluctuate. EPCs near to multiples of 8 pips (especially 16) have a higher winning chance than expected.

Any comments are appreciated.

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Appendix

Exact values for (5)

a0 = 4.56
a1 = 1.95
a2 = 1.31
a3 = 0.84
a4 = 0.40
a5 = 0.23
a6 = 0.11

Exact values for (7)

EPC	ΔGWC / ΔEPC
104.7	2.0
74.5	2.5
56.4	3.0
44.6	3.5
36.3	4.0
30.4	4.5
25.9	5.0
22.4	5.5
19.4	6.0

Special thanks

... to the GnuBG team (www.gnubg.org) for providing their bearoff database and a nice position to eps export.

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... to the owners of Regression (www.numericalmathematics.com) for the free program that helped doing most of the necessary mathematical calculations.

... to amarganth (FIBS) and tryout (FIBS) for rereading the item and making useful suggestions.

Changes

Pages 2 + 6 "If there are checkers on the acepoint and at least 4 points are made, subtract 1 pip." replaced by "If there is at least 1 checker on the acepoint and at least 4 points are made, subtract 1 pip."