Backgammon Funfair

Major new content in Edition 2 (November 2012) compared with Edition 1 (May 2012)

Chapter 22 is new (two pages).

Two new pages at the start of Chapter 24 (previously Chapter 23). This chapter is now entitled: *Lowest non-zero chance to win*

The previous title was: *Lowest non-zero chance to win without hitting*. The new pages discuss no-contact positions.

These new pages are below, after a blank page.

Viewing with two pages open is recommended (just as in the book).

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22

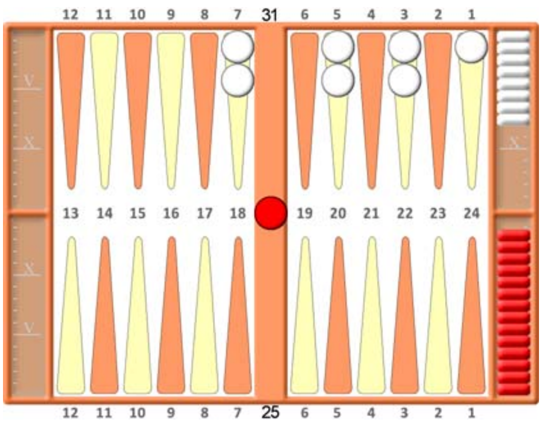
Fewest rolls which hit direct shots because of the *higher number* rule

In Chapters 16 to 21, the fewest rolls which hit direct shots result from the rule that Red has to play the numbers on both dice if possible.

Another rule is that, if Red can play either number but not both, he must play the higher number. Below are the fewest rolls which hit direct shots because of this rule.

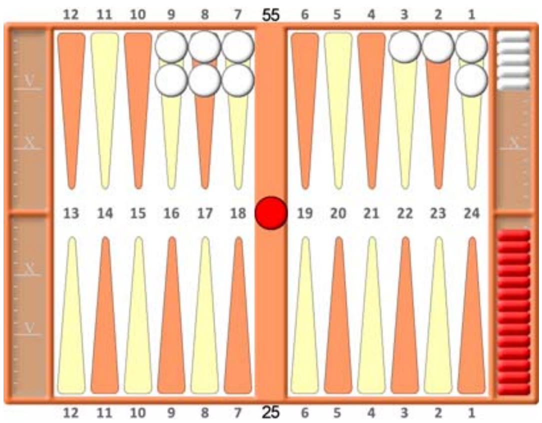
For convenience Red's checker is on the bar. However in each case Red's checker could be on his 24pt with each of White's checkers moved back one pip; or on his 23pt with each of White's checkers moved back two pips; and so on.

One and two direct shots



Red on roll has **one** direct shot with a 1 and only **five** hits

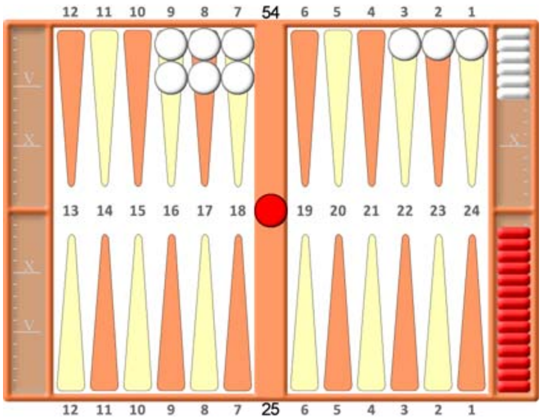
Red misses with 61, 41 and 21



Red on roll has a **double** direct shot with 3 and 2 and only **ten** hits

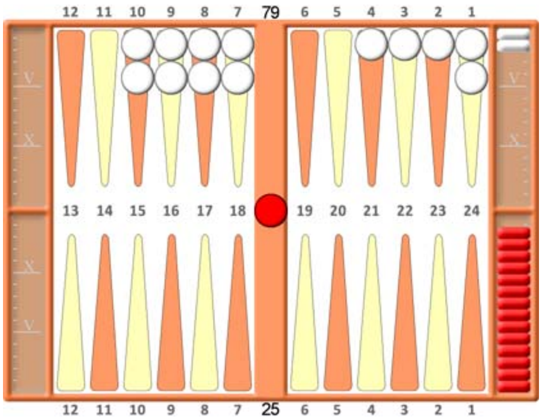
Red misses with 63, 53, 43; and with 62 and 52

Three direct shots



Red on roll has a **triple** direct shot with 3, 2 and 1 and only **fifteen** hits

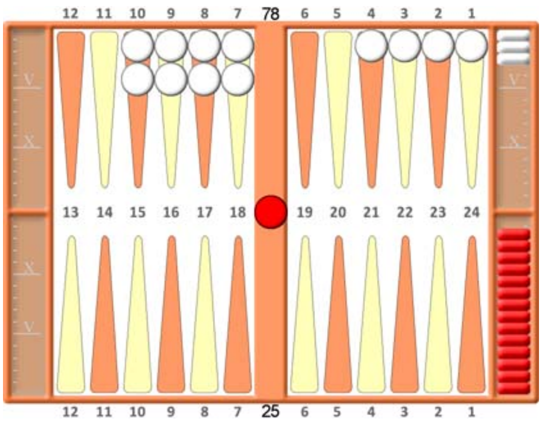
Red misses with 63, 53 and 43; 62 and 52; and with 61



Red on roll has a **triple** direct shot with 4, 3 and 2 and only **fifteen** hits

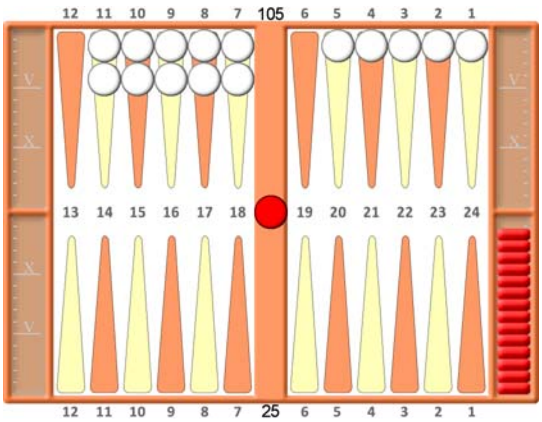
Red misses with 64 and 54; 63 and 53; and with 62 and 52

Four and five direct shots



Red on roll has a **quadruple** direct shot with 4, 3, 2 and 1 and only **eighteen** hits

Red misses with 64 and 54; 63 and 53; 62 and 52; and with 61



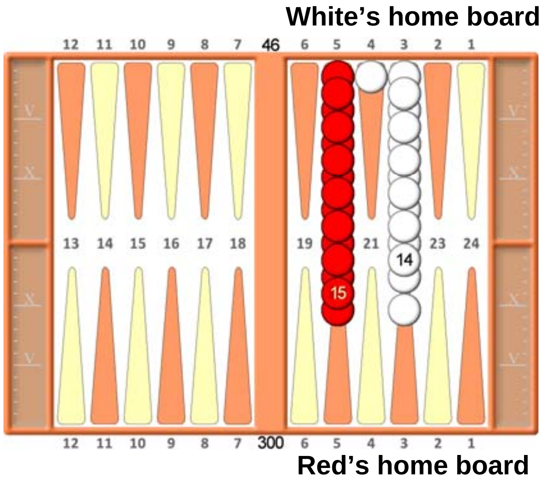
Red on roll has a **quintuple** direct shot with 5, 4, 3, 2 and 1 and only **twenty-five** hits

Red misses with 65, 64, 63, 62 and 61

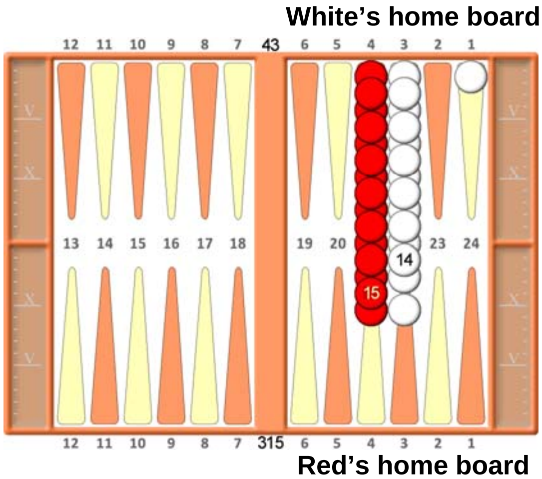
24

Lowest non-zero chance to win

Red can win in each of these **no-contact** positions. Position 1 has the lower chance to win but it is quite close.



Position 1
White on roll



Position 2
Red on roll

Maximum rolls required by White to bear off

Sixteen First fifteen rolls are each 21 and last roll is anything	Fifteen First fourteen rolls are each 21 and last is anything Trivial variation: White checker on her 1pt is instead on 2pt or 3pt

Minimum rolls required by Red to bear off

Fifteen rolls of doubles sufficient to get four cross-overs on each roll	Fifteen rolls of doubles sufficient to get four cross-overs on each roll

How can Red win?

Red rolls fifteen doubles which get four cross-overs on each roll, while White has fifteen rolls of 21	Red rolls fifteen doubles which get four cross-overs on each roll, while White has fourteen rolls of 21

What is the probability that Red wins?

(A) Probability that White rolls 21 fifteen times is $(2/36)^{15}$	(A) Probability that White rolls 21 fourteen times is $(2/36)^{14}$
(B) Fifteen rolls of 66, 55, 44, 33 or 22 can arrive in 5^{15} = 30,517,578,125 orders. Of these, 107,713,585 have the required 300 or more pips.[1]	(B) Fifteen rolls of 66, 55, 44, 33 or 22 can arrive in 5^{15} = 30,517,578,125 orders. Of these, 6,913,880 have the required 315 or more pips.[1]
If each of these orders could secure four cross-overs, the probability that Red bears off with fifteen rolls would be 107,713,585/(36^{15}) ...	If each of these orders could secure four cross-overs, the probability that Red bears off with fifteen rolls would be 6,913,880/(36^{15}) ...
... and the probability that Red wins would be (A)*(B) = $7.22*10^{-35}$ (thirty-four zeros after decimal point)	... and the probability that Red wins would be (A)*(B) = $8.34*10^{-35}$ (thirty-four zeros after decimal point)

For the purpose of achieving a lower probability for Red, (A) is better for Position 1 while (B) is better for Position 2. Multiplying (A) and (B), Position 1 has the lower probability.

Many of the orders of fifteen Red dice rolls with sufficient pips do not secure four cross-overs on each roll. For example, nine 66s, three 55s, one 33 and two 22s have 304 pips and can arrive in 300,300 orders. How many orders work? Answering that requires a program to move the checkers optimally.

My guess is that the four cross-overs constraint will reduce the probability that Red wins by more in Position 1 than Position 2. This is because Position 1 has many more orders with sufficient pips. If this guess is correct, the difference between Red's true winning probabilities in the two positions will widen, with Position 1 having the lower value.

[1] E-mail to request Excel file ***Lowest non-zero chance to win***. A roll of 11 fails because Red can never be on his 19pt, 13pt, 7pt or 1pt without already having missed a cross-over on a previous roll.